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VOLUME XLII DECEMBER 1915 NUMBER 5

THE SPECTROSCOPIC DETERMINATION OF THE SOLAR ROTATION AT OTTAWA

By J. S. PLASKETT

INTRODUCTION

The value of the solar rotation obtained from plates made at Ottawa in 1911 has already been published[1] by the writer in collaboration with R. E. DeLury. The present paper gives a summary and discussion of the measures of plates obtained in 1912 and 1913. It was deemed desirable, for the sake of homogeneous discussion and comparison of the results for the three years, to publish my measures separately from DeLury's, which are not yet completed and will appear later. It may be mentioned at the outset that the differences found to exist between DeLury and myself in the measures of the same plates in 1911 still persist in the same direction, though slightly reduced in magnitude.

The same apparatus and methods used in obtaining the 1911 plates, described fully elsewhere,[2] were employed in making the 1912 and 1913 plates. The plates of 1912 were made by DeLury and the writer jointly, while those of 1913 were made by DeLury and

[1] *Astrophysical Journal*, **37**, 73, 1913, and *Transactions Royal Society of Canada*, 1912, Sec. III, p. 1.

[2] *Report of Chief Astronomer*, 1910, p. 129, and *Transactions Royal Society of Canada*, 1911, Sec. III, p. 107.

my son, H. H. Plaskett, during my absence at the Solar Union Meeting at Bonn. The quality for measurement of the 1911 plates is considerably superior to those of 1912 and these latter again are better than the 1913. This difference is due in the main to a change in the emulsion of the plates used, the 1911 lot being especially sensitive and fine-grained, and also, partly, to greater care being used in the earlier plates in regard to development to obtain the most suitable density for measurement. Furthermore a greater number were made in 1911, and the best of these being selected for measurement also tended to a better average of quality. It must be stated, however, that equal care and precautions to avoid all danger of systematic error were taken in 1912, and I believe in 1913, as in 1911, that the difference in quality refers only to the ease and accuracy of measurement of the plates. As will be seen later, the accidental errors of measurement of all these plates are small in comparison with the systematic differences involved.

All the 1912 and 1913 plates were measured on a Repsold Measuring Engine, the small differential quantities involved being obtained with its eyepiece micrometer, whose run was carefully and frequently determined by comparison with the attached calibrated scale. The multipliers required to reduce the displacements to velocities were obtained by measurements of the spectra with this scale. In the 1911 plates four settings were made on a line in the center strip and two each on the corresponding line in the two outer strips, and, after all the lines had been measured, the plate was reversed on the engine and the measurement repeated. In the 1912 and 1913 plates, however, a reversing prism was placed on the eyepiece, and, after two settings on the center strip and one each on the outside strips had been made, the prism was rotated through 90° and the same number of settings made on the apparently reversed position of the plate, the settings direct and reversed being completed on one line before passing to the next. Hence only half the number of settings were made on the later plates, thus reducing by half the large amount of measuring required.

The labor entailed in these measurements was further reduced by diminishing the number of lines measured. It will be remem-

bered that one of the subjects for investigation decided upon at the Solar Union Meeting in 1910 was to determine whether different lines or different elements gave systematic differences of velocity, and the large number of lines used in 1911, 19 at λ 5600 and 15 at λ 4250, were selected so as to include as many elements as possible. However, the results obtained in 1911 seemed to be so decisive negatively, especially at λ 5600, that it was not felt worth while to continue this investigation. Hence the number of lines measured at λ 5600 was reduced to 12 in 1912 and still further reduced to 6, except in the equatorial plates, in 1913. In the λ 4250 region, however, the 15 lines were measured in 1912, though this number was reduced to 7 in 1913.

The purpose of thus reducing both the number of lines and the number of settings on a line was to enable more plates to be measured with the same amount of labor. This reduction was justified on the one hand by the failure to find any systematic differences of velocity for different elements and on the other by the fact that the measures of the 1911 plates showed that the probable error of measurement of a plate, determined from the internal agreement of the lines on that plate, was less than one-fifth of the probable error of a plate as determined from the agreement of the plates among themselves. Hence a determination of the rotation obtained by doubling the number of plates and halving the number of lines measured on a plate would be of much greater weight. Even if no greater number of plates were used, the accidental error of measurement of a plate in which the number of lines measured was reduced to one quarter would still be less than half the plate errors and could not have an appreciable effect on the resulting values, while the lessening of the labor of measurement would be a decided advantage.

Of the plates obtained in 1912, 25 were selected in the λ 5600 region and 25 in the λ 4250 region for measurement. On the λ 5600 plates spectra at eight latitudes, 0°, 15°, 30°, 45°, 60°, 75°, 80°, 85°, were made and measured, and on the λ 4250 plates the first six of these latitudes only. Of the 1913 plates 20 in the λ 5600 region at the same eight and 17 in the λ 4250 region at the same six latitudes were measured.

The lines measured in these plates, the elements to which they belong, their intensity in Rowland's tables, and the multiplier to reduce the measured double displacement in millimeters to the velocities of the observed point on the limb in kilometers per second are given in Tables I and II.

TABLE I

LINES IN λ 5600 REGION

No.	Wave-Length	Elem.	Int.	Velocity Constant	No.	Wave-Length	Elem.	Int.	Velocity Constant
1.....	5544.157	Fe	2	19.118	* 7....	5582.198	Ca	4	18.899
2.....	5560.434	"	2	19.024	* 8....	5590.343	"	3	18.852
3.....	5562.933	"	2	19.010	* 9....	5601.505	"	3	18.788
*4.....	5569.848	"	6	18.970	10....	5624.769	Fe-V	3	18.653
*5.....	5576.320	"	4	18.933	11....	5638.488	Fe	3	18.575
*6.....	5578.946	Ni	1	18.919	12....	5658.097	Y	2	18.461

TABLE II

LINES IN λ 4250 REGION

No.	Wave-Length	Elem.	Int.	Velocity Constant	No.	Wave-Length	Elem.	Int.	Velocity Constant
1.....	4196.699	La	2	26.906	* 9....	4257.815	Mn	2	26.400
2.....	4197.257	C	2	26.902	*10....	4258.455	Fe	2	26.394
3.....	4216.136	"	1	26.745	11....	4266.081	Mn	2	26.331
*4.....	4220.509	Fe	3	26.710	12....	4268.015	Fe	2	26.296
*5.....	4225.619	"	3	26.666	13....	4276.836	Zr	2	26.243
*6.....	4232.887	"	2	26.606	14....	4290.377	Ti	2	26.133
*7.....	4241.285	Fe-Zr	2	26.502	15....	4291.630	Fe	2	26.122
*8.....	4246.996	Se	5	26.490	....	....	......	....	....

In the λ 5600 region the 12 lines were used in the measures of all the 1912 plates and in the 1913 plates at the equator, but in the 1913 higher latitudes and in repeated equatorial measures of all three years only the six starred lines were used. In the λ 4250 region the 15 lines were used in all the 1912 plates, but only the 7 starred ones in the 1913 plates and in repeated equatorial measures of all three years.

In the measures of the 1911 plates two methods of reducing the measured values to the actual velocities were used, which, however, as was to be expected, gave practically identical results.

Consequently in the 1912 and 1913 plates only one of these methods has been used, that which was called the first method in the earlier paper and which depends upon the projection of the observed points to the limb, the obtaining of the corrections by

TABLE III

SUMMARY OF MEASURES, 1912, λ 5600

PLATE NUMBER	DATE G.M.T.	VELOCITIES IN KM PER SECOND AT LATITUDE REGIONS							
		0°	15°	30°	45°	60°	75°	80°	85°
	June								
885	5.20	2.055	1.927	1.648	1.178	0.766	0.402	0.286	0.094
886	6.17	2.019	1.925	1.636	1.245	.857	.292	.193	.148
887	6.20	2.013	1.873	1.637	1.207	.831	.277	.157	.132
888	6.24	1.974	1.866	1.637	1.243	.827	.326	.211	.116
889	8.10	1.956	1.882	1.605	1.233	.835	.420	.236	.141
890	8.12	2.027	1.890	1.639	1.219	.799	.440	.261	.117
893	8.24	1.994	1.846	1.718	1.256	.831	.405	.288	.120
898	13.16	2.041	1.906	1.659	1.237	.808	.371	.223	.115
899	13.22	2.047	1.908	1.655	1.275	.807	.398	.216	.084
900	13.35	2.027	1.900	1.639	1.271	.795	.397	.235	.125
901	13.37	2.043	1.874	1.640	1.223	.806	.437	.288	.128
903	14.18	1.969	1.896	1.628	1.257	.702	.359	.251	.116
907	17.19	1.978	1.848	1.605	1.252	.804	.385	.230	.122
908	17.21	1.961	1.866	1.641	1.234	.681	.396	.210	.147
909	17.23	2.013	1.907	1.625	1.217	.725	.379	.208	.158
923	26.21	2.006	1.852	1.578	1.270	.794	.332	.262	.145
924	26.23	1.978	1.893	1.579	1.219	.796	.367	.239	.154
925	26.25	1.997	1.842	1.601	1.252	.818	.350	.246	.145
926	26.30	2.024	1.782	1.557	1.186	.898	.351	.275	.133
927	26.32	2.026	1.827	1.572	1.188	.754	.318	.263	.149
928	26.34	2.023	1.854	1.603	1.182	.833	.352	.280	.155
929	26.36	2.026	1.871	1.663	1.179	.937	.354	.262	.145
930	27.10	2.051	1.851	1.641	1.196	.806	.396	.243	.116
931	27.12	2.050	1.897	1.628	1.216	.813	.347	.271	.144
932	27.15	2.060	1.866	1.604	1.196	0.764	0.351	0.231	0.157
Mean linear velocity		2.014	1.874	1.627	1.225	0.803	0.368	0.243	0.132
Mean angular velocity		14°.31	13°.77	13°.34	12°.29	11°.38	10°.03	9°.79	10°.15
Mean latitude		0°0′	15°0′	29°59′	44°58′	59°57′	74°57′	79°51′	84°.42
Probable error plate		0.022	0.022	0.022	0.022	0.031	0.026	0.022	0.013

Dunér's methods and tables, and the application of a final correction, necessary owing to the distance of the observed point from the limb, to allow for the change of rotation with latitude.

MEASURES OF PLATES

However desirable it may be to publish the individual velocities of the lines on the plates, it will be evident that this cannot be done here, when it is realized that 222 spectra were measured on

TABLE IV

Summary of Measures, 1912, λ 4250

Plate Number	Date G.M.T.	Velocities in Km per Second at Latitude Regions					
		0°	15°	30°	45°	60°	75°
	October						
940	5.14	1.950	1.900	1.662	1.214	0.776	0.292
942	5.19	2.025	1.875	1.745	1.210	.790	.370
943	5.23	2.030	1.962	1.671	1.248	.851	.388
944	5.25	2.048	1.918	1.672	1.249	.825	.409
948	8.20	2.013	1.931	1.564	1.260	.790	.462
949	8.23	1.957	1.919	1.562	1.207	.782	.452
950	8.25	2.022	1.868	1.639	1.193	.824	.371
952	12.14	1.979	1.875	1.609	1.269	.874	.325
953	12.16	2.002	1.952	1.628	1.282	.854	.300
954	12.18	2.081	1.864	1.633	1.263	.820	.357
955	12.20	2.078	1.896	1.714	1.217	.831	.367
956	12.24	1.982	1.967	1.700	1.235	.845	.372
957	12.27	1.974	1.962	1.774	1.338	.845	.416
959	16.11	2.019	1.871	1.697	1.291	.823	.406
960	16.14	2.031	1.932	1.699	1.318	.792	.392
961	16.17	2.058	1.922	1.670	1.270	.811	.364
962	16.19	2.089	1.910	1.649	1.275	.746	.427
963	16.22	2.052	1.948	1.634	1.234	.736	.403
964	16.24	1.996	1.914	1.667	1.247	.829	.433
965	16.26	2.025	1.885	1.723	1.251	.806	.446
966	16.28	1.952	1.880	1.729	1.207	.847	.432
967	17.17	1.919	1.896	1.660	1.469	.991	.498
968	17.20	1.931	1.939	1.659	1.390	1.033	.401
969	17.22	1.934	1.951	1.719	1.341	.896	.321
970	17.25	1.970	1.888	1.748	1.329	0.905	0.323
Mean linear velocity		2.005	1.914	1.673	1.272	0.832	0.389
Mean angular velocity		14°.24	14°.06	13°.70	12°.70	11°.63	9°.96
Mean latitude		0°0′	14°55′	29°52′	44°42′	59°29′	73°54′
Probable error plate		0.035	0.024	0.035	0.040	0.037	0.034

the 1911 plates, 350 on the 1912, and 262 on the 1913, making a total of 834 spectra measured and reduced. The total number of lines measured was over 11,000 and of settings over 120,000.

Hence all that will be given here are the reduced velocities for all latitudes on all the plates of 1912 and 1913 with the number

and Greenwich mean date of the plates. The mean linear and mean daily angular velocities at the mean latitudes are given at the foot of the columns with the probable errors of single plates.

Table VII contains a summary of the mean values for the different latitudes for the four series in 1912 and 1913 and also for the

TABLE V

SUMMARY OF MEASURES, 1913, λ 5600

PLATE NUMBER	DATE G.M.T.	VELOCITIES IN KM PER SECOND AT LATITUDE REGIONS							
		0°	15°	30°	45°	60°	75°	80°	85°
	June								
979....	6.30	2.031	1.940	1.645	1.225	0.815	0.365	0.206	0.125
985....	9.23	1.961	1.841	1.647	1.257	.899	.397	.209	.113
986....	9.25	1.953	1.903	1.678	1.351	.819	.439	.218	.061
987....	10.20	2.017	1.894	1.560	1.167	.823	.391	.243	.135
988....	10.21	2.011	1.879	1.600	1.206	.788	.390	.239	.178
989....	10.24	2.017	1.795	1.630	1.226	.840	.408	.204	.170
990....	10.25	2.034	1.905	1.626	1.253	.773	.395	.293	.145
992....	10.27	2.019	1.841	1.628	1.176	.795	.350	.219	.200
994....	10.30	2.008	1.895	1.620	1.234	.753	.322	.218	.108
996....	10.32	2.008	1.912	1.642	1.249	.740	.377	.244	.146
998....	10.35	1.979	1.887	1.651	1.222	.836	.403	.231	.142
1000....	10.37	1.986	1.890	1.674	1.299	.785	.328	.246	.128
1003....	13.22	1.999	1.940	1.679	1.233	.798	.306	.267	.105
1005....	13.26	1.954	1.880	1.648	1.244	.781	.313	.243	.151
1007....	13.28	1.930	1.913	1.653	1.197	.792	.353	.283	.114
1011....	15.23	1.964	1.889	1.580	1.226	.819	.347	.290	.117
1013....	15.26	1.939	1.839	1.590	1.192	.810	.357	.278	.112
1015....	15.28	1.963	1.904	1.554	1.221	.853	.393	.261	.138
1017....	15.31	2.015	1.917	1.539	1.219	.805	.392	.275	.076
1018....	15.32	1.998	1.942	1.567	1.232	0.823	0.362	0.268	0.124
Mean linear velocity		1.989	1.891	1.621	1.237	0.807	0.369	0.247	0.129
Mean angular velocity		14°.11	13°.90	13°.29	12°.36	11°.46	10°10	10°.07	10°.34
Mean latitude ...		0°0′	15°0′	30°0′	45°0′	60°0′	74°58′	79°58′	84°55′
Probable error plate		0.022	0.022	0.030	0.022	0.022	0.025	0.020	0.020

two series in 1911. When each of these values is reduced by the final mean formula to the even latitudes and the weighted mean for each latitude taken, we have the mean values in the last column of the table. The velocities are expressed in kilometers per second.

DISCUSSION OF VALUES

In discussing and comparing the values we notice first of all that the total range of velocity for any one latitude in any series obtained from Tables III–VI is rather higher than would be expected, the average total range being 0.157 km per second, the smallest being 0.074 and the largest, 0.297. Further, it can be seen from Table VII that the mean values for each latitude, except

TABLE VI

SUMMARY OF MEASURES, 1913, λ 4250

Plate Number	Date G.M.T.	Velocities in Km per Second at Latitude Regions					
		0°	15°	30°	45°	60°	75°
	June						
1030	25.22	1.971	1.942	1.684	1.274	0.852	0.392
1031	25.30	1.924	1.906	1.669	1.268	.866	.406
1032	27.15	1.932	1.869	1.638	1.214	.877	.406
1035	28.17	2.001	1.844	1.545	1.142	.798	.361
	July						
1050	11.20	1.919	1.892	1.581	1.207	.740	.380
1055	13.10	2.089	1.901	1.742	1.275	.782	.451
1056	16.14	2.009	1.931	1.625	1.182	.814	.450
1061	17.35	1.938	1.839	1.534	1.195	.823	.375
1063	18.34	1.979	1.777	1.702	1.223	.847	.336
1065	19.18	1.920	1.863	1.675	1.230	.819	.343
1073	25.18	1.995	1.882	1.671	1.284	.846	.438
1076	26.13	2.009	1.827	1.648	1.348	.790	.367
1077	26.16	1.974	1.880	1.644	1.214	.777	.385
1078	26.19	1.952	1.817	1.593	1.267	.804	.408
1081	28.32	1.929	1.901	1.738	1.218	.876	.351
	August						
1091	11.15	1.958	1.919	1.544	1.318	.939	.394
1092	11.20	2.003	1.888	1.650	1.286	0.816	0.405
Mean linear velocity		1.971	1.876	1.640	1.244	0.828	0.391
Mean angular velocity		14°.00	13°.78	13°.42	12°.44	11°.62	10°.22
Mean latitude		0°3′	14°54′	29°50′	44°45′	59°36′	74°14′
Probable error plate		0.029	0.029	0.042	0.037	0.031	0.023

the two highest (80° and 85°), vary over a total range of between 0.040 and 0.052, but that these differences do not run regularly: the series having high values at the equator may have low values at the higher latitudes and vice versa. The run of these differences seems to be in the main accidental, and there is no indication of any regular or systematic law governing them.[1] It will be

[1] A possible exception to this may be pointed in that the angular values at 80° are systematically considerably lower than at 85°.

noticed further that both series in 1913 have lower values at the equator than in 1911 and 1912, although at the higher latitudes this difference is not maintained; for example, in 1912, λ 5600 has

TABLE VII

SUMMARY OF MEAN VALUES

Year	1911		1912		1913		Mean Values Reduced to Even Latitude
Region	λ 5600	λ 4250	λ 5600	λ 4250	λ 5600	λ 4250	
Number of Spectra	19	24	25	25	20	17	
Latitude	0°2′	0°0′	0°0′	0°0′	0°0′	0°3′	0°0′
Linear velocity..	2.017	2.012	2.014	2.005	1.989	1.971	2.003
Angular velocity.	14°.32	14°.28	14°.30	14°.23	14°.11	14°.00	14°.20
Latitude	15°0′	....	15°0′	14°55′	15°0′	14°54′	15°0′
Linear velocity..	1.886	....	1.874	1.914	1.891	1.876	1.889
Angular velocity.	13°.86	....	13°.78	14°.00	13°.90	13°.78	13°.88
Latitude	29°58′	29°59′	29°59′	29°52′	30°0′	29°50′	30°0′
Linear velocity..	1.652	1.625	1.627	1.673	1.621	1.640	1.639
Angular velocity.	13°.54	13°.32	13°.33	13°.70	13°.29	13°.42	13°.41
Latitude	44°52′	....	44°58′	44°42′	45°0′	44°45′	45°0′
Linear velocity..	1.273	....	1.225	1.272	1.231	1.244	1.246
Angular velocity.	12°.75	....	12°.29	12°.70	12°.36	12°.44	12°.51
Latitude	59°46′	59°53′	59°57′	59°29′	60°0′	59°36′	60°0′
Linear velocity..	0.809	.788	.803	.832	.807	.828	.805
Angular velocity.	11°.41	11°.15	11°.38	11°.63	11°.46	11°.62	11°.43
Latitude	74°28′	....	74°54′	73°54′	74°58′	74°14′	75°0′
Linear velocity..	0.417	....	.368	.389	.369	.391	.373
Angular velocity.	11°.05	....	10°.03	9°.96	10°.10	10°.22	10°.23
Latitude	79°53′	....	79°51′	....	79°58′	....	80°0′
Linear velocity..	0.247	....	.243	....	.247	....	.247
Angular velocity.	9°.98	....	9°.79	....	10°.07	....	10°.10
Latitude	84°47′	....	84°42′	....	84°55	....	85°0′
Linear velocity..	0.131	....	.132	....	.129	....	.127
Angular velocity	10°.23	....	10°.15	....	10°.34	....	10°.34

values generally lower at the higher latitudes than both series in 1913, though its equatorial value is considerably higher.

In order to smooth out these irregularities for each series and in the mean, it will be desirable to obtain the formulae connecting the change of velocity with latitude. Formulae of the Faye type

have been generally used, not only for the spectroscopic determinations, but also in the case of rotation determined from sun-spots, faculae, and flocculi. These formulae seem to represent the observations within the limiting accuracy of the values obtained.

In the present case the following forms have been employed, as they have the advantage that the a and a' constants give the equatorial velocities directly.

$$\text{Linear velocity, } V=(a-b\sin^2\phi)\cos\phi$$
$$\text{Angular velocity, } \xi=a'-b'\sin^2\phi$$

Owing to difficulty in suitably weighting the different latitudes, the angular formulae have been obtained directly from the linear by substitution. The constants which are tabulated in Table VIII have been obtained by least squares by combining the observations in three different ways:

1. Each of the six series of plates have had independent constants computed.

2. The two series in a year have been combined, thus giving three yearly constants.

3. All the observations in the three years have been combined, giving the mean Ottawa constants.

TABLE VIII

CONSTANTS OF FAYE FORMULAE

Year	Region	Linear		Angular	
		a	b	a'	b'
1911	λ 5600	2.012	0.500	14°28	3°55
	λ 4250	2.016	.587	14.31	4.17
	Both Regions	2.012	.518	14.29	3.68
1912	λ 5600	2.002	.558	14.21	3.96
	λ 4250	2.022	.514	14.36	3.64
	Both Regions	2.012	.541	14.28	3.84
1913	λ 5600	1.993	.518	14.14	3.68
	λ 4250	1.980	.470	14.06	3.34
	Both Regions	1.988	.498	14.11	3.54
1911–12–13	All Plates	2.006	0.522	14.24	3.71

The differences in the constants above given for the 1911 plates from those previously published are due, in the linear values, to limiting the rotation results to those obtained by the first method of reduction; and, in the angular values, to substituting from the linear constants instead of computing directly.

Although these constants, especially the b and b', may seem to vary considerably, we find that they may be altered over quite wide limits without appreciably increasing the residuals between the observed velocities and those computed by the formulae. Table IX gives the residuals between the observed velocity at each

TABLE IX

RESIDUALS IN KILOMETERS FROM FORMULAE

Latitude	Observed Velocity	Residuals		
		Independent	Yearly	Mean
1911 λ 5600				
0° 2′	2.017	+0.005	+0.005	+0.011
15 0	1.886	− .024	− .024	− .018
29 58	1.652	+ .017	+ .021	+ .027
44 52	1.273	+ .024	+ .030	+ .036
59 46	.809	− .016	− .007	− .001
74 28	.417	+ .002	+ .007	+ .012
79 53	.247	− .022	− .018	− .018
84 37	.131	− .006	− .005	− .003
1911 λ 4250				
0 0	2.012	− .004	± .000	+ .006
29 59	1.625	+ .006	− .006	± .000
59 53	.788	− .003	− .026	− .018
1912 λ 5600				
0 0	2.014	+ .012	+ .002	+ .008
15 0	1.874	− .024	− .034	− .030
29 59	1.627	+ .014	+ .001	+ .002
44 58	1.225	+ .004	− .007	− .011
59 57	.803	+ .010	− .002	− .006
74 54	.368	− .018	− .025	− .027
79 51	.243	− .014	− .019	− .021
84 42	.132	−0.001	−0.004	− .005
Means				
0 0	2.003	....	....	− .003
15 0	1.889	....	....	− .015
30 0	1.639	....	....	+ .015
45 0	1.246	....	....	+0.012

Latitude	Observed Velocity	Residuals		
		Independent	Yearly	Mean
1912 λ 4250				
0° 0′	2.005	−0.017	−0.007	−0.001
14 55	1.914	− .005	+ .005	+ .009
29 52	1.673	+ .031	+ .044	+ .045
44 42	1.272	+ .016	+ .032	+ .033
59 29	.832	− .001	+ .014	+ .011
73 54	.389	− .040	− .031	− .033
1913 λ 5600				
0 0	1.989	− .004	+ .001	− .017
15 0	1.891	± .000	+ .002	− .012
30 0	1.621	+ .007	+ .008	− .003
45 0	1.231	+ .007	+ .002	− .003
60 0	.807	+ .005	± .000	± .000
74 58	.369	− .022	− .027	− .025
79 58	.247	− .013	− .016	− .014
84 55	.129	− .001	− .003	− .002
1913 λ 4250				
0 3	1.971	− .009	− .017	− .035
14 54	1.876	− .006	− .013	− .029
29 50	1.640	+ .023	+ .023	+ .013
44 45	1.244	+ .003	+ .008	+ .004
59 36	.828	+ .003	+ .009	+ .010
74 14	.391	−0.027	−0.024	− .022
Means				
60 0	.805	....	....	− .002
75 0	.373	....	....	− .020
80 0	.247	....	....	− .014
85 0	0.127	....	....	−0.002

latitude in each series and the velocity computed from the three sets of constants, the independent, the yearly, and the general mean. It is at once seen that the differences between these are relatively very small in each series, and it may safely be said that all the Ottawa observations are satisfactorily represented and that the law of variation of velocity with latitude is given by the formulae

$$\text{Linear velocity, } V=(2.006-0.522 \sin^2 \phi) \cos \phi$$
$$\text{Daily angular velocity, } \xi=14^\circ\!.24-3^\circ\!.71 \sin^2 \phi$$

Returning now to the discussion of the differences in rotation values for 1913 and 1911 and 1912, and the irregularities in the mean values for the same latitudes, it was previously pointed out that the former consisted chiefly in a diminished value, about 2 per cent, at the equator. When the irregularities were partially smoothed out by the formulae, this was reduced to about 1 per cent. Table IX shows that, although the mean formula represents all the series without introducing residuals abnormally large, there still remain these residuals and the irregularities in the mean values, as well as the large ranges in the individual values, to be accounted for.

CAUSES OF DIFFERENCES

These differences and irregularities may be attributed to one or more of three causes:

1. Systematic instrumental errors in the plates obtained.

2. A change in the habit or personal equation of measurement of the observer.

3. A change or changes in the rate of rotation of the sun.

1. *Systematic instrumental errors.*—This question was quite fully discussed in the earlier paper and not much need be added to what was there said. The four essential precautions to prevent systematic instrumental displacements, which were there given and which have been scrupulously followed throughout the observations, may be here repeated.

a) The emulsion on the photographic plate must be exactly in the focus of the spectrum.

b) The illumination of the grating from the opposite limbs of the sun must be similar and uniform.

c) The solar definition must be good, the image steady, and the sky free from haze.

d) Care must be taken that the reflecting prisms receive light from the desired and supposed latitudes.

The focus was most carefully determined repeatedly both by the test and by the Hartmann extra-focal method and could never have been sufficiently out to produce any appreciable displacement even if condition *b* was not exactly fulfilled. Nevertheless the reflecting prisms in front of the slit were always adjusted before each plate was exposed so that the circles of illumination from the two limbs were exactly superposed and central over the exposed part of the grating surface, which was only about half the diameter of these circles. The illumination was always examined after the exposures were finished to see that no change had occurred.

Owing to the large size, 225–230 mm, of the solar image given by the Ottawa coelostat telescope considerable advantage so far as condition *d* is concerned prevailed over other observers of the solar rotation where the images have been considerably smaller; and the ease and accuracy with which the spectrograph can be rotated to any desired position angle around the optical axis facilitated the exact setting to and determining of the solar latitudes observed.

Furthermore, the practice, first adopted here, I think, in the case of the solar rotation, of placing a narrow strip of spectrum from the one limb of the sun between two strips of exactly the same width from the other limb, completely eliminates the possibility of error arising, in the case of only one strip from each limb, from the micrometer wire not being parallel to or remaining parallel to the lines of the spectrum.

So far as can be seen, therefore, the probability of any systematic instrumental error in these observations is slight. Independent confirmation of this is given by the fact that observations made in 1910[1] with a different grating and other varying experimental conditions gave a mean velocity at the equator for ten plates of 2.011 km per second, almost exactly the same as in 1911 and 1912. When a possible change in the habit of measurement is considered we have the four years in excellent agreement.

[1] *Report of Chief Astronomer*, 1910, p. 129.

2. *Personal equation of measurement.*—That the same plates measured by different individuals may give values differing by 2 per cent is evident from the previously published paper, where Plaskett-DeLury was on the average about 0.040 km per second. There were also curious differences in measures of the same plates between the Ottawa and Mt. Wilson observers. The rotation value is hence uncertain to the extent of the personal equation of the measurer and it was felt desirable to carry on further work to attempt to clear up this elusive difference.

Suggestions as to the cause of a personal equation in measurement have been advanced by H. H. Plaskett[1] who suggests that a more or less permanent habit of setting is formed, partly due to following the path of least resistance, the method of setting in which the least mental energy is required, and partly to a possibly unconscious prepossession of the mind looking for a certain result. In such a case it is possible that if a sufficient interval elapsed between measures the habit of measurement might change. Evidence of the probability of this in my own case is given below in remeasures of all plates at the equator.

A comparison of the measured values of the 1912 plates between DeLury and myself shows that the differences are in the same direction as in 1911, only slightly reduced in magnitude. Since the earlier paper DeLury has measured all the λ 5600 plates and all the equator spectra in the λ 4250 region in 1911, as well as the 25 plates each in λ 5600 and λ 4250 in 1912. In the following table the mean values of the differences for all these measures are given and also the differences J.S.P.–H.H.P. for 5 plates, λ 5600, in 1912, and the 20 plates, λ 5600, in 1913:

TABLE X

DIFFERENCES IN MEASURES OF SAME PLATES

Year	Region λ	No. of Plates	Observers	0°	15°	30°	45°	60°	75°	80°	85°
1911...	5600	19	J.S.P.–R.E.D.	+0.045	+0.025	+0.036	+0.038	+0.022	+0.020	....	....
" ...	4250	24	" – "	+ .024	....	....	....	....	....	....	....
1912...	5600	25	" – "	+ .008	+ .014	+ .009	+ .002	+ .017	+ .025	+0.033	+0.012
" ...	4250	25	" – "	+ .041	+ .029	+ .038	+ .036	+ .041	+ .025	....	....
" ...	5600	5	" –H.H.P.	+ .021	+ .021	+ .014	+ .011	+ .001	+ .001	+ .019	– .007
1913...	5600	20	" –H.H.P.	+0.000	+0.054	+0.018	+0.006	+0.015	+0.003	+0.004	–0.003

[1] *Journal R.A.S.C.*, 7, 307, 1914.

These differences, which are in kilometers per second, run in a curious way. If we take J.S.P.–R.E.D. we find in 1911 at λ 5600 a mean difference of +0.031 and at λ 4250, +0.024. In 1912 at λ 5600 the mean difference is +0.015 and at λ 4250, +0.035, a curious reversal of the order of magnitude. The yearly difference decreases from 0.028 to 0.025 and the mean of the whole is 0.027 km per second. The mean differences J.S.P.–H.H.P. in 1912 is +0.010 while in 1913 it is +0.012. If, however, we omit the 15° latitude 1913, about which there is evidently something abnormal in one or both of the measures, the mean difference diminishes to +0.006.

The difference J.S.P.–R.E.D., though varying in different regions and different years, is persistent and evidently systematic, with an average magnitude of +0.027 km per second. On the contrary the difference J.S.P.–H.H.P. is much smaller, diminishes, and shows a tendency to vanish altogether.

These differences and the smaller rotation values for the 1913 plates led me to remeasure all the equatorial plates of the three years. In order to avoid chance of prepossession. all the λ 5600 plates were arranged at random and the year and number of the plate not identified until after the remeasurement, and the same procedure was followed with the λ 4250 plates. Hence a homogeneous and directly comparable set of measures of the 130 equatorial plates were obtained. The mean results are given in the following table:

TABLE XI

REMEASURES OF EQUATOR PLATES

Year	λ 5600 Region				λ 4250 Region			
	No. Plates	Original	Remeasure	Diff.	No. Plates	Original	Remeasure	Diff.
1911	19	1.824	1.806	0.018	24	1.754	1.739	0.015
1912	25	1.848	1.835	.013	25	1.774	1.755	.019
1913	20	1.807	1.805	0.002	17	1.743	1.742	0.001

This remeasurement was made almost directly after the measures of the 1913 plates, and it will be seen that their measures have not changed, but that the remeasured values of the 1911 and 1912 plates average 0.016 km per second lower than the original.

If a correction of this amount is applied it will bring the plates of all four years 1910–13 into remarkably good agreement. A change in habit of measurement is hence sufficient to account for the difference between 1913 and 1911–12.

It is not possible to say, when these variations in measures of the same plates are present, what to assign as the true value. It may be stated as having some bearing that the writer's probable error of measurement is somewhat less than H. H. Plaskett's and only about half that of DeLury's. It would seem probable, therefore, considering this and the differences in Tables X and XI, that his remeasured values are not far from the truth and this would make the equatorial rotational value about 2 km per second. However, considering that nothing definite can be said, I prefer to leave the mean value as obtained above, and given by the mean formulae, especially as the differences are evened up in this way and as it cannot certainly be said that the remeasured values are superior to the original.

3. *Change in the rate of rotation of the sun.*—The considerations adduced in the last section, which bring all the values at Ottawa over four years into remarkably close agreement, form strong evidence against any general variation of the rate of rotation. In this connection it may be useful to give a summary of the values obtained by the spectroscopic method elsewhere, consequently they are tabulated below. It must be remembered that in some of the values tabulated the constants are not given in this form in the original papers and have been roughly computed from the data. The values may hence in some cases be subject to slight corrections, but will, however, serve for comparison.

The large range shown in these values of the rotation does not necessarily indicate a variation in the rate, as they were made by different observers with great diversity in instrumental equipment and methods of observing and measuring. The measures seem to group themselves generally into three sets: (1) high values: Dunér, Halm, Adams, and Storey and Wilson, averaging about 2.06 km per second at the equator; (2) low values: Hubrecht and Evershed and Royds, with an average equatorial value of 1.90 km; (3) intermediate values: Schlesinger and Ottawa, 2.00 km.

Halm considered that he had discovered a periodicity in the solar rotation, but it seems doubtful whether these early observations were of sufficient accuracy to permit of definite conclusions. The probable errors of Dunér's and Halm's visual observations were several times greater than Adams' values, the internal agreement of which was exceedingly satisfactory. The probable errors of Storey and Wilson's, of Hubrecht's, and of Evershed and Royds's plates are considerably greater than those given by the Mt. Wilson and Ottawa plates, and it seems probable that the exceptionally low value of Hubrecht is due to some instrumental or measuremental error. Even excluding the low group, however, there still

TABLE XII

Constants of Faye Formulae—Other Observers

Observer	Linear		Angular	
	a	b	a'	b'
Dunér	2.08	0.59	14.81	4.2
Halm	2.05	.39	14.53	2.5
Adams, 1906–07	2.055	.48	14.59	3.4
Adams, 1908	2.05	.55	14.61	4.0
Storey and Wilson	2.08	.45	14.75	3.2
Hubrecht	1.86	.45	13.23	3.2
Evershed and Royds	1.94	....	13.77	....
Schlesinger	2.00	.48	14.17	3.4
DeLury, 1911	1.97	.52	14.00	3.7
H. H. Plaskett, 1913	1.98	.54	14.04	3.8
J. S. Plaskett, 1911–13	2.01	.52	14.24	3.7

remains the difference between Mt. Wilson 2.05 km and Ottawa and Allegheny 2.0 km to be accounted for. I do not believe that this difference is due to a change in the velocity of rotation of the sun, but to plate or measurement errors in one or both of the groups.

The mean value of the solar rotation as determined from sun-spots by Carrington, Spoerer, and Maunder is just midway between the Ottawa and Mt. Wilson values, but determinations from faculae and flocculi are nearer in agreement to the latter. However, the velocity of the reversing layer is not necessarily the same as that of these visible phenomena.

I understand that the rate of rotation is now being redetermined at Mt. Wilson with the 150-ft. tower telescope and the 75-ft. grating spectroscope. The use of this unequaled equipment, combined with

the experience and skill of the Mt. Wilson observers, should give values of great weight, and it will be of much interest to see with which group of observations they most nearly agree.

While believing that the general velocity of the reversing layer is not subject to change, there seems to be no other means of accounting for the large differences in the measured values of plates taken under apparently identical conditions, for the high ratio of plate error to measurement error, and for the variation in the mean values at the different latitudes, than to assume them to be due to local movements or eddy currents in the reversing layer of a comparatively transitory character. Such movements are known to exist around sun-spots and have been detected in other regions, both at Mt. Wilson and Ottawa, of magnitudes sufficient to readily account for the observed differences.

PROBABLE ERRORS

In Table XIII have been compiled both measurement and plate errors for the three years. On the first line we have the probable errors of measurement of single lines and on the second the measurement error of a plate obtained by dividing the foregoing values by the square root of the number of lines measured on the plate. The last line contains the mean plate errors obtained by comparing the plates in each series.

TABLE XIII

PROBABLE ERRORS

Kind of Error	1911		1912		1913	
	λ 5600	λ 4250	λ 5600	λ 4250	λ 5600	λ 4250
Measurement—single line	0.024	0.015	0.030	0.031	0.040	0.045
Measurement—single plate	.006	.004	.009	.008	.016	.017
Total single plate........	0.028	0.026	0.023	0.034	0.039	0.032

We find from the foregoing table by comparing the last two lines that the measurement error varies from about one-sixth to one-half the total error of a plate, the higher ratios occurring where the number of lines and settings had been markedly diminished. Indeed it is seen that the total errors are not much different

from the measurement error of a single line and that hence the use of only three or four lines on each plate would be amply sufficient.

Some additional information in this direction is given by the remeasurement of the equatorial plates of all series, a comparison of the total plate errors for each measurement being given in this table.

TABLE XIV

PROBABLE ERRORS OF EQUATORIAL PLATES

TOTAL PLATE ERRORS	1911		1912		1913	
	λ 5600	λ 4250	λ 5600	λ 4250	λ 5600	λ 4250
Original measure.........	0.013	0.018	0.021	0.031	0.021	0.028
Remeasurement..........	0.024	0.023	0.026	0.029	0.028	0.024

Although only 6 or 7 lines were used in the remeasures as compared with 12 to 19 in the original measures, there is not much difference in the plate errors, showing that the measurement effects are relatively small and that the cause of the relatively high total probable error of a plate as compared with the computed measurement error is to be sought in differences in the plates themselves, caused probably, as previously inferred, by local temporary disturbances in the reversing layer.

That the measurement error of a plate determined directly is in good agreement with that obtained by dividing the probable error of a line by the square root of the number of lines, is shown by the comparison of six remeasures of plate 867 of the 1911 λ 4250 series at three latitudes, 0°, 30°, 60°, 12 lines being measured on each spectrum, the results of which are given in the following table:

TABLE XV

PROBABLE ERRORS OF REMEASURES OF PLATE 867

	0°	30°	60°
Mean probable error, single line................	±0.018	±0.024	±0.011
Mean probable error, plate = above ÷ $\sqrt{12}$.......	.0052	.0069	.0032
Probable error, plate from comparison of 6 measures	±0.0057	±0.0077	±0.0034

SYSTEMATIC DIFFERENCES OF VELOCITY FOR DIFFERENT ELEMENTS

This question was fully discussed in the 1911 measures where the conclusion was reached that no systematic difference for different lines or elements was present. The mean algebraic residual for any line was in no case greater than one-third the mean numerical residual. The same lines were measured in 1912, λ 4250, as in 1911, λ 4250, and the residuals were similarly tabulated with a similar result. It was concluded that no difference in velocity for different lines or elements, which cannot readily be explained by accidental errors of measurement, is present in the Ottawa plates.

CONCLUSIONS

The conclusions following from the measures of the Ottawa plates 1910 to 1913 by the writer may be summarized as follows:

1. The value of the solar rotation determined at Ottawa can be expressed by the formulae

$$V=(2.006-0.522 \sin^2 \phi) \cos \phi$$
$$\xi = 14.^\circ 24-3.^\circ 71 \sin^2 \phi$$

2. The values for the four years in which observations were obtained are in excellent agreement and the values of the constants of the foregoing formulae obtained from each of the six series are also in good agreement. The differences in the residuals, using the mean and separate values of the constants, are negligibly small, and the mean constants satisfactorily represent all Ottawa observations.

3. So far as the interval covered by the Ottawa observations, 1910 to 1913, inclusive, is concerned, the solar rotation is constant. The slight decrease obtained in 1913 is shown to be very probably due to a change in the habit of measurement.

4. The total range of mean velocity in each latitude is about 0.04 km per second. The most probable explanation of such differences is to ascribe them to local motions of the gases in the reversing layer.

5. The errors of measurement of the plates average about one-fourth the total errors as obtained by comparison of the plates, the average of the latter for a single plate being about 0.03 km per second.

6. Personal differences of measurement of different observers may be very much greater than the probable measurement error of either. The difference J.S.P.–R.E.D. averages +0.027 km per second and J.S.P.–H.H.P. +0.008 km per second.

7. The personal equation of measurement may change, the remeasurement of the equator plates of 1911 and 1912 giving values about 0.015 km per second lower.

8. In consideration of the possibility of these personal differences and changes, the value of the rotation may be uncertain to the extent of one or two hundredths of a kilometer. Such personal effects may explain part of the differences in the values obtained by different observers.

9. All the spectroscopic observations of the rotation of the reversing layer may be grouped into three sets of values, high, medium, and low. High values are obtained by Upsala, Edinburgh, and Mt. Wilson, and average at the equator 2.06 km per second. Medium values, obtained at Allegheny and Ottawa, run about 2.00 or 2.01 km. Low values, obtained at Cambridge and Kodaikanal, average about 1.9 km. The cause of such large differences in the values can most probably be assigned mainly to observational or instrumental errors and secondarily to personal differences of measurement.

10. The large number of plates and measures made at Ottawa, much greater than at any other observatory, their close interagreement over four years' interval, and the care employed in the making and measurement of the plates entitle the Ottawa value to considerable weight.

11. No indication of any systematic differences of velocity for different lines or elements is given by the Ottawa measures.

I have much pleasure in expressing my appreciation of the readiness of the Director, Dr. W. F. King, to supply the necessary apparatus and of his encouragement and interest throughout.

Dominion Observatory, Ottawa
June 1915

THE TRANSPARENCY OF AQUEOUS VAPOR[1]

By F. E. FOWLE

The chief object of this communication is to treat quantitatively of the depletion of energy from the radiation of heavenly bodies as it passes through atmospheric aqueous vapor. In earlier communications[2] the non-selective depletion or *scattering* was treated; in this, the selective depletion or *absorption* will be considered. Further, a summary will be given relating to atmospheric absorption in general between the wave-lengths $0.35\,\mu$ and $2.00\,\mu$, and formulae and data for determining it for any given case.

THE NON-SELECTIVE SCATTERING

The non-selective scattering of energy varies continuously with the wave-length and is easily expressed as a continuous function of the wave-length. In the case of the permanent gases of the atmosphere above Mount Wilson on clear days the scattering is almost purely molecular and may be computed from the number of molecules present in the path. In the case of water vapor, the losses are considerably greater than would be expected from purely molecular scattering and are apparently caused by grosser particles associated with the water vapor. The scattering varies so slowly with the wave-length that the coefficients which express it depend but slightly upon the purity of the spectrum. Coefficients of non-selective depletion, $a_{a\lambda}$ and $a_{w\lambda}$, for dry, dust-free air and water vapor, respectively, have been published. As just stated, these vary slowly with the wave-length. In order to know the intensity of the energy after transmission through the air, it is necessary only to multiply the original intensity of the energy from the heavenly body at the wave-length λ by $a_{a\lambda}^{m} a_{w\lambda}^{mw}$, where the exponents m and w express the length of path and the amount of

[1] Published by permission of the Secretary of the Smithsonian Institution.

[2] *Astrophysical Journal*, **38**, 392, 1913; **40**, 435, 1914.

water vapor.[1] The transmission coefficients for dry air and aqueous vapor will be found in Table I (p. 403).

The coefficients of transmission for dust will be considered only incidentally in this paper. Above an altitude of 1000 meters dust is generally negligible on clear days. At sea-level the dust coefficients are very variable from day to day. They are probably nearly the same for all wave-lengths less than 3 μ. The average scattering caused by the dust above Washington on clear days is about 9 per cent. On one of the clearest days on which observations have been made there it amounted to 3 per cent (February 15, 1907).[2]

THE SELECTIVE ABSORPTION

Selective absorption presents a very different case. It exists practically only in bands at certain wave-lengths, and within these bands varies very rapidly. If absorption or transmission coefficients were determined for these bands, as in the earlier case, the values would depend greatly upon the purity of the spectrum. For instance, in Fig. 1, if *ab* is an absorption band in a pure spectrum, the transmission would be taken as bc/ac; in an impure spectrum for the same quantity of vapor it would wrongly appear to be, say, $b'c/ac$. It can be shown, however, that the areas of the bands remain nearly the same in the two cases. The areas, being nearly independent of the purity, therefore, have been utilized as a measure of the absorption. Unfortunately Bouguer's formula cannot then be used,[3] as with the scattering coefficients, and the results must be expressed empirically.

In each water-vapor band the absorption at first increases rapidly with increasing wave-lengths to a maximum, and then dies away more slowly. From this lack of symmetry, it results that the

[1] m is taken as unity when the body is in the zenith. It is equal to the secant of the zenith distance to within 1 per cent when the zenith distance is less than 70°. For greater zenith distances than 70°, Bemporad's air masses for the exponent of $a_{a\lambda}$ should be used, while retaining sec z for the exponent of $a_{w\lambda}$ because of the low level of the water vapor. See *Smithsonian Miscellaneous Collections*, **65**, No. 4, 1915, and *Mitteilungen der Grossherzoglichen Sternwarte zu Heidelberg*, No. 4, 1904. w is the depth in centimeters of the precipitable water in the atmosphere above the place.

[2] *Meteorologische Zeitschrift*, **6**, 270, 1914; *Monthly Weather Review*, **42**, 2, 1914.

[3] *Annals of the Astrophysical Observatory of the Smithsonian Institution*, **2**, 16, 1908.

equality of areas just considered holds only for radiation in similar spectra, that is, from sources of the same order of temperatures. Thus, referring to Fig. 1, had the energy-curve the slope xy, the area of the band would in general have been different, even in a pure spectrum, though the amount of incident energy distributed over the wave-lengths under consideration were the same. Accordingly due consideration must be employed in applying the results given below to the cases of radiation from bodies at terrestrial and laboratory temperatures. The results as here obtained apply with their *full* accuracy only to a distribution of radiation-energy such as is found in this region in the solar spectrum.

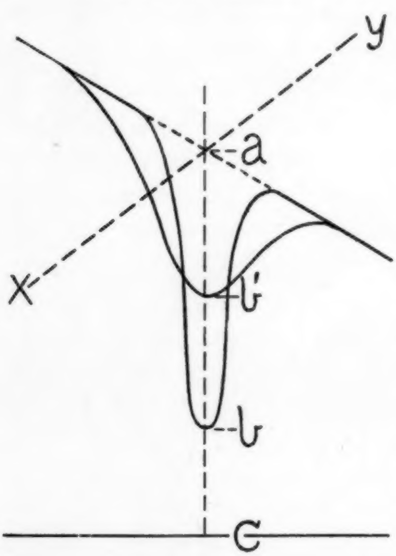

FIG. 1

The various atmospheric bands, the absorption in which will be considered, as well as the contour of the solar energy-curve, are given in Fig. 2. This spectrum lies between the wave-lengths 0.65 μ and 2.12 μ. The coefficients of the general atmospheric transmission vary so little from wave-length to wave-length here that the general contour of the energy-curve, neglecting the bands, scarcely varies from day to day, or with the time of the day, either without or within the atmosphere. About 95 per cent of the radiation sent to us from the sun lies in this region between 0.30 μ and 2.40 μ.[1] The radiation from a body at the mean temperature of the earth, 287° K. = 14° C., lies in quite a different region in the spectrum and is affected by entirely different series of absorption bands having wave-lengths greater than 2 μ. Quantitative measures of the absorption by water vapor in this latter region are in progress here. For a body at the temperature of the

[1] The selective absorption in the region near the D lines is comparatively small and probably nearly wholly taken into account by the general transmission coefficients.

Nernst lamp 60 per cent of the radiation lies at wave-lengths greater than those of the region treated in this communication.

There are two quantities the measurement of which is to be considered: the amount of water vapor and the corresponding absorption of energy passing through it.

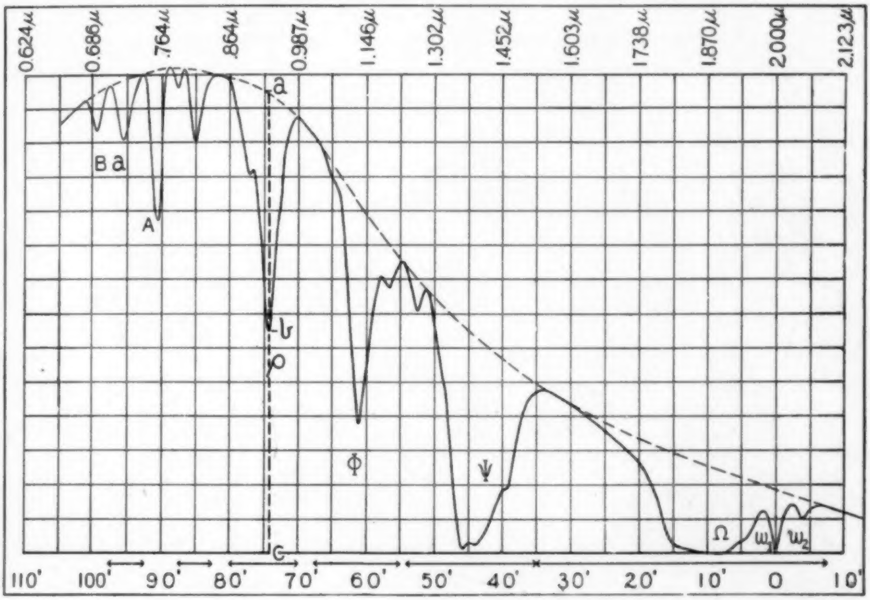

FIG. 2.—Contour solar energy-curve, 60° ultra-violet glass prism. Region of atmospheric absorption bands.

MEASUREMENT OF THE AMOUNT OF WATER VAPOR

A spectroscopic method for determining the amount of atmospheric water vapor and the applications of the method have already been given.[1] The ratio of the deflection in the bottom of the water-vapor bands, ρ and Φ, to the ordinates of the smooth curve drawn across the tops of the bands, was observed with known amounts of water vapor in the laboratory. The largest layer of water vapor used was equivalent to about 0.6 cm of precipitable water, and the curve connecting the ratios for ρ and Φ with the amount of water vapor was produced to about 3 cm precipitable water vapor,

[1] *Astrophysical Journal*, **35**, 149, 1912; **37**, 359, 1913.

guided by certain conditions stated in the first of the abovementioned articles. These curves are used to determine the amounts of water vapor producing the absorptions treated in this communication. For instance, in Fig. 2 the ratio of *ab*, the intensity of energy in ρ, to *ac* is 0.48 and from curve *a* in Fig. 3 we find this corresponds to 2.0 cm of precipitable water vapor.

The following results lead to increased confidence in the validity of this relationship between the depths of these bands and the amount of water vapor. Sounding balloon ascensions were made from Avalon during the summer of 1913 by Mr. Sherry of the United States Weather Bureau and Mr. Aldrich of this Observatory.[1] Avalon is on the Santa Catalina Island, off the southern coast of California, about 60 miles to the southwest of Mount Wilson. Observations were made spectroscopically on Mount Wilson on three days on which balloon ascensions were made and furnish the following comparisons:

Dates	July 23	August 3	August 8
Spectroscopic values..	1.17 cm	2.06 cm	1.39 cm
Balloon values......	1.07 cm	2.09 cm	1.41 cm

The agreement is very satisfactory considering that the observations differed slightly in time and place and that the direct laboratory calibration extended only to 0.6 cm precipitable water, whereas these comparisons necessitated the use of the calibration-curve to over 6 cm (because some of the observations with the spectroscope were made through air masses three times that at the zenith).[2]

[1] *Monthly Weather Review*, **42**, 410, 1914.

[2] The estimation of the amount of water vapor, whether right or wrong, introduces no error into the solar-constant determinations made here. The absorption values used in those reductions depend upon the relationship between the measured depths of ρ and the area of the absorption bands which is experimentally determined without regard to the amount of water vapor. However, as we have the relationship between the depth of ρ and the precipitable water, the results seem more intelligible if the depth of ρ is eliminated between the two functions, and the absorption expressed in terms of the amount of water vapor in the air. In Fig. 3 are given the curves by which the deflections in ρ and Φ may be interpreted in terms of the precipitable water. Curves *a* and *c* are for ρ and Φ with a purity of spectrum such that the bolometer-plus-the-slit width is equal to 0.022 μ, and curves *v* and *d* when it is equal to 0.0057 μ. Linear interpolation between these two curves may be used to construct plots for intermediate purities.

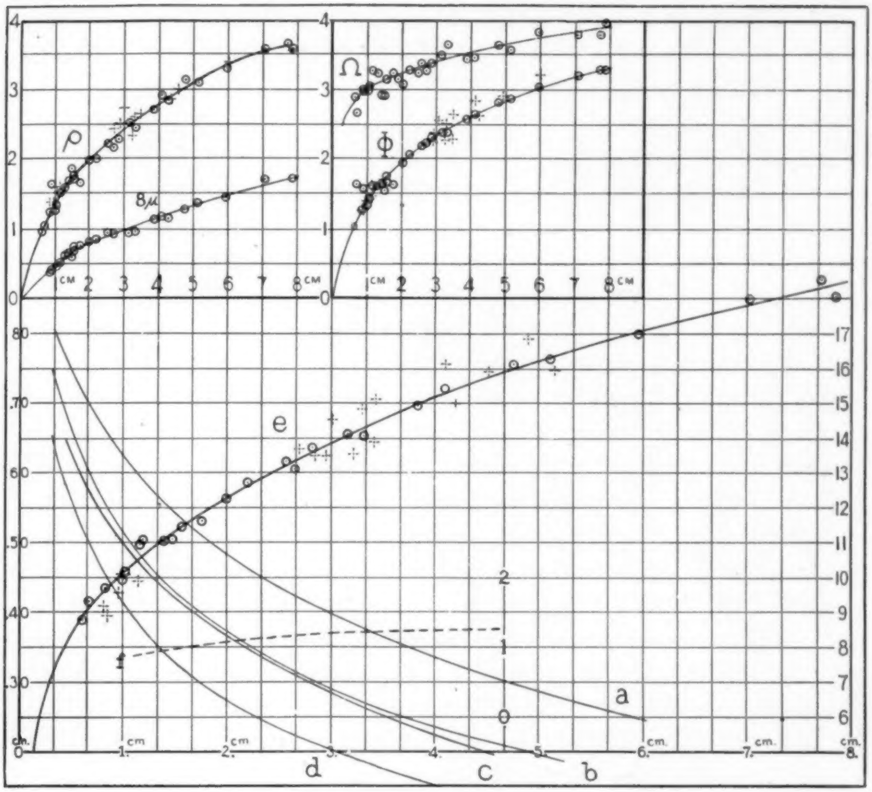

FIG. 3.—Curves for areas of bands, etc.

Curves marked ρ, 0.8 μ, Ω, Φ, and *e:*

Abscissae are precipitable water in cm.

Ordinates are areas of absorption bands. The first four are for separate bands; curve *e* is the sum of all the bands. The circles are Mount Wilson data; the crosses, Washington values with different prism and purity of spectrum, but less accurate because of smaller scale.

Curves marked *a*, *b*, *c*, and *d:*

Abscissae are precipitable water in cm.

Ordinates are ratios of depths of bands to top.

a and *c* are for ρ and Φ respectively, slit+bolometer=0.022 μ

b and *d* are for ρ and Φ respectively, slit+bolometer= 0.0057 μ.

Curve marked *f:*

Abscissae, for cm read air masses.

Ordinates, number 0, 1, 2 in the center of the plot, are the areas of the atmospheric bands other than those due to water vapor in the same units as curve *e*.

MEASUREMENT OF THE SELECTIVE ABSORPTION IN CONNECTION WITH DETERMINATIONS OF THE SOLAR RADIATION

The second quantity, the atmospheric absorption of radiation, as dependent upon the amount of aqueous vapor, will now be considered. At Mount Wilson (altitude 1730 m; barometer 62.3 cm) curves are obtained showing how this absorption is distributed in the spectrum of the solar energy after this energy has passed through the air. There was shown in Fig. 2 the red and infra-red end of such an energy-curve, indicating the principal absorption bands produced by the constituents of the atmosphere. The following list gives the bands, their wave-lengths, and the constituent of the air producing them:

	μ			μ	
B	0.69	Oxygen	Φ	1.13	Water vapor
a	.72	Water vapor	Ψ	1.42	Water vapor
A	.76	Oxygen	Ω	1.89	Water vapor
—	.81	Water vapor	ω_1	2.01	?
$\rho\sigma\tau$	0.93	Water vapor	ω_2	2.05	?

Simultaneously with obtaining these curves the pyrheliometer is read. Evidently the total area included by these energy-curves, suitably corrected for instrumental losses, may be placed equal to the number of calories indicated by the pyrheliometer, thus obtaining the *scale* or the number of calories per square centimeter of the energy-curve or bologram. Whence it would be easy to convert the areas of the bands, which represent their absorption, into the number of calories absorbed by the atmospheric vapors and gases.

For solar-radiation computations the area included by the smoothed energy-curve (i.e., including all the band areas) is obtained by adding together the measured ordinates of the smoothed curve taken at equal intervals. It would be tedious to measure directly the areas of all the bands with satisfactory accuracy; but it has been found feasible to express their total area as a function of the depth, or fractional depression from the smoothed energy-curve, at ρ or Φ. This quick method requires the assumption that the form of the smoothed energy-curve in the infra-red spectrum region, where the atmospheric bands occur (see Fig. 2),

should be constant from hour to hour and from day to day. Investigation has shown that its small changes of form, which depend principally on the variations of atmospheric transparency, do not affect substantially the accuracy of the process, as applied to energy-curves taken with moderate air masses (less than 5) at Bassour, Mount Wilson, or Mount Whitney.

Accordingly the areas of the bands have been thus determined, suitably corrected for instrumental losses, multiplied by the reciprocal of the smooth curve's ordinate in centimeters at ρ (*ac*, Fig. 2).[1] They can then be expressed accurately as a simple function of the amount of water vapor as measured by the depth of ρ. Several curves showing such results are given in Fig. 3. In the upper part of the figure are given the values of the areas for ρ, the band at 0.8 μ for Ω, and for Φ. Curve *e* gives the sum of the areas of all the bands produced by water vapor. Had the general transmission in the infra-red been more variable with the wave-length, such a simple function of the water vapor would not have been right. A set of similar curves would have been necessary for each air mass, and had the transmission also varied from day to day, even these would not have been sufficient.

Returning to the energy-curve of Fig. 2, we found from the depth of ρ that the amount of water vapor producing the deflection was 2.0 cm. The curve for ρ and that marked *e* give, for this amount of vapor, for the band ρ and the total area of all the water-vapor bands, areas of 1.96 and 12.3 sq. cm, respectively. Assuming that each square of Fig. 2 represents 1 sq. cm, then since the whole ordinate at ρ is 13.4 cm and the scale of abscissae (2 cm to 10′) is twice the standard scale, the corresponding areas for Fig. 2 should

[1] This is in order to reduce to standard conditions which are taken so that the smooth curve's ordinate at $\rho = 10$ cm. It may be desirable to use the curves for getting the absorption in a solar spectrum produced by apparatus of different dispersion. In such cases, since the areas of the bands remain the same although the ordinates change, it will be necessary to multiply the absorption areas taken from the plots, not only by the smoothed-curve ordinate in centimeters at ρ, but also by the ratio of the old to the new dispersion at ρ. The area of the bands in square centimeters is given in curve *e*, Fig. 3, when the prismatic solar energy-curve (dispersion at $\rho = 0.12\ \mu$ to 10′ of deviation) is plotted on a scale of 1 cm to 10′ of deviation and the ordinate of the smoothed curve above ρ equals 10 cm. The curve connecting the depth of ρ with the precipitable water must be corrected as already indicated for the purity of the spectrum (e.g., bolometer+slit-widths) expressed in microns (μ).

be $2 \times 1.34 \times (1.96$ and $12.3)$, or about 5.25 and 33.0 sq. cm, respectively. The scale in calories would be obtained through a pyrheliometer reading and the areas of absorption then converted into calories.

ABSORPTION DUE TO THE PERMANENT GASES OF THE AIR

There is another set of bands which must be taken into account; those due to oxygen and those of unknown atmospheric origin. These bands all seem to vary only with the length of path or air mass (which is very nearly equal to the secant of the zenith distance for zenith distances less than 70°). Accordingly these bands have been measured and reduced to the same standard scale, and their sum appears in curve f of Fig. 3. For use with an observed energy-curve, the areas taken for the proper amount of water vapor in the one case, and the proper air mass in the other, are multiplied by the observed ordinate of the smoothed curve above ρ. Curve f is given for Mount Wilson. An air mass m_x at another altitude where the barometer reads x cm is equal to that at Mount Wilson multiplied by the ratio of x to 62.

ATMOSPHERIC LOSSES IN CALORIES AT MOUNT WHITNEY, MOUNT WILSON, AND WASHINGTON

In what has preceded, the selective absorption has been discussed as actually determined and used in reducing the solar radiation bolograms made here, at Mount Wilson, and elsewhere. With the other atmospheric losses, it will now be given under two somewhat different forms: first, as the number of calories absorbed from the incoming 1.93 calories of the solar radiation under certain concrete conditions; secondly, as the fractional transmission of the incident energy by water vapor between certain wave-lengths.

The atmospheric losses from the incoming solar energy comprise five parts: (1) that due to the general scattering by the molecules of the permanent gases of the atmosphere; (2) that due to the general scattering associated with water vapor; (3) that due to selective (banded) absorption of the permanent gases of the atmosphere; (4) that due to the selective (banded) absorption of water vapor; (5) that due to dust.

Table I contains the wave-lengths in μ, the intensities of the solar energy outside the atmosphere, $e_{o\lambda}$, given on an arbitrary, absolute scale of intensities, but on the relative scale from wave-length to wave-length of a 60° ultra-violet glass prism; the atmospheric transmission coefficients for dry air, barometer 623 mm,

TABLE I

60° Ultra-Violet Glass Prismatic Solar Energy-Curve; Also Dry Air and Aqueous Vapor (1 cm Precipitable Water) Transmission Coefficients

λ.......	0.342	0.350	0.360	0.371	0.384	0.397	0.413	0.431	0.452	0.475
$e_{o\lambda}$......	102	130	160	198	227	322	437	518	681	807
$a_{a\lambda}$.....	(0.595)	(0.626)	0.655	0.686	0.713	0.752	0.783	0.808	0.840	0.863
$a_{w\lambda}$.....	0.920	0.926	0.934	0.940	0.945	0.949	0.953	0.957	0.961	0.964
λ.......	0.503	0.535	0.574	0.624	0.686	0.764	0.864	0.987	1.146	1.302
$e_{o\lambda}$.....	907	1044	1197	1334	1416	1435	1431	1306	1025	775
$a_{a\lambda}$.....	0.885	0.898	0.905	0.929	0.959	0.979	0.987	0.992	0.996	0.997
$a_{w\lambda}$.....	0.968	0.972	0.970	0.975	0.981	0.984	0.986	0.987	0.987	0.987
λ.......	1.452	1.603	1.738	1.870	2.000	2.123	2.242	2.348	......	......
$e_{o\lambda}$.....	586	435	343	262	187	123	88	74	......	......
$a_{a\lambda}$.....	0.998	0.999	0.999	0.999	0.999	0.999	0.999	0.999	......	......
$a_{w\lambda}$.....	0.987	0.987	0.987	0.987	0.986	0.985	0.984	0.983	......	......

altitude 1730 meters, $a_{a\lambda}$, and the transmission coefficients for 1 cm of precipitable water vapor, $a_{w\lambda}$. In order to determine the absolute "scale," the area of the energy-curve constructed with the foregoing $e_{o\lambda}$ data has been placed equal to the mean value of the solar constant, 1.93 calories[1] per square centimeter per minute at the mean solar distance of the earth. Then with use of the formula

$$e_\lambda = e_{o\lambda}\{a_{a\lambda}a_{w\lambda}^{w}\}^m$$

energy-curves have been constructed for various air masses and amounts of water vapor, the absorption data of this paper being employed.

Tables II–IV resulted from measures of areas from these curves. They have been compared and found to be consistent with energy-curves actually observed. The first triplet of lines in each of the three tables gives, first, the amount of radiation scattered from the direct solar beam by the permanent gases of the atmosphere;

[1] The calorie used is the 15° C. gram-calorie, or the amount of heat necessary to warm one gram of water 1° C. at 15° C.

secondly, the amount selectively absorbed by them (B, A, a, ω_1, ω_2 lines); and, thirdly, the sum of these two quantities. For the succeeding triplets (except for Washington) are given, first, the additional amount scattered by the water vapor; secondly, the amount selectively absorbed by it (a, $\rho\sigma\tau$, etc., lines); and, thirdly, the totals in which for each case are included the totals from the first triplets. For the Washington values, in the lines designated "water scattered," is included the amount scattered by the dust. The amounts lost by dust, at the zenith, on the three days included in the table are respectively, February 15, 3 per cent, October 4, 9 per cent, and May 14, 14± per cent (see also *Monthly Weather Review* or *Meteorologische Zeitschrift, op. cit.*). The data for the Mount Whitney and Mount Wilson tables have been computed directly from data of Table I by means of the formula just given. Except for the first

TABLE II

MOUNT WHITNEY. ATMOSPHERIC ABSORPTION FOR DRY AIR AND DRY AIR PLUS VARIOUS AMOUNTS OF WATER VAPOR

Altitude 4420 m; Barometer 44.7 cm

Incident solar radiation, 1.93 15° C.-gram-calories per sq. cm per minute

Air Masses	$m=1$		$m=2$		$m=3$		$m=4$		$m=5$		$m=7$	
Precipitable Water Vapor	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost
0.00 cm												
Air scattered	0.14	7.3	0.23	11.9	0.31	16.1	0.38	19.7	0.44	22.8	0.55	28.5
Air absorbed	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.02	1.0
Total lost	.15	8.0	.24	12.0	.32	17.0	.39	20.0	.45	23.0	.57	30.0
0.11 cm												
H_2O scattered	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.02	1.0	.02	1.0
H_2O absorbed	.08	4.1	.10	5.2	.11	5.7	.12	6.2	.12	6.2	.13	6.7
Total lost	.24	12.0	.35	18.0	.44	23.0	.52	27.0	.59	31.0	.72	37.0
0.25 cm												
H_2O scattered	.01	.5	.02	1.0	.03	1.6	.04	2.1	.04	2.1	.05	2.7
H_2O absorbed	.10	5.2	.12	6.2	.13	6.7	.14	7.3	.15	7.8	.16	8.3
Total lost	.26	14.0	.38	20.0	.48	25.0	.57	30.0	.64	33.0	.78	40.0
0.50 cm												
H_2O scattered	.02	1.0	.04	2.1	.06	3.1	.07	3.6	.08	4.1	.10	5.2
H_2O absorbed	.12	6.2	.15	7.8	.16	8.3	.17	8.8	.18	9.4	.20	10.4
Total lost	0.29	15.0	0.43	22.0	0.54	28.0	0.63	33.0	0.71	37.0	0.87	45.0

triplet of lines for dry, dust-free air, a somewhat different procedure has been followed for the Washington data. For Washington a third coefficient $a_{d\lambda}$ would be necessary within the brackets of the formula just given to take into account the scattering by the dust. Instead of computing for various values $a_{a\lambda}$ and $a_{w\lambda}$ and $a_{d\lambda}$, it was thought best to make computations using the actually observed product of these three coefficients for three days of widely different conditions.[1]

TABLE III

MOUNT WILSON. ATMOSPHERIC ABSORPTION FOR DRY AIR AND DRY AIR PLUS VARIOUS AMOUNTS OF WATER VAPOR

Altitude 1730 m; Barometer 62.3 cm

Incident solar radiation, 1.93 15° C.-gram-calories per sq. cm per minute

Air Masses	$m=1$		$m=2$		$m=3$		$m=4$		$m=5$		$m=7$	
Precipitable Water Vapor	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost
0.00 cm												
Air scattered	0.15	7.8	0.28	14.5	0.38	19.7	0.47	24.4	0.54	28.0	0.66	34.2
Air absorbed	.01	0.5	.01	0.5	.01	0.5	.02	1.0	.02	1.0	.02	1.0
Total lost	.16	8.0	.29	15.0	.39	20.0	.49	25.0	.56	29.0	.68	35.0
0.33 cm												
H_2O scattered	.02	1.0	.02	1.0	.04	2.1	.04	2.1	.04	2.1	.05	2.6
H_2O absorbed	.11	5.7	.13	6.7	.14	7.3	.15	7.8	.16	8.3	.17	8.8
Total lost	.29	15.0	.44	23.0	.57	30.0	.68	35.0	.76	39.0	.90	47.0
0.50 cm												
H_2O scattered	.03	1.6	.03	1.6	.06	3.1	.06	3.1	.07	3.6	.08	4.1
H_2O absorbed	.12	6.2	.15	7.8	.16	8.3	.17	8.8	.18	9.3	.19	9.8
Total lost	.31	16.0	.47	24.0	.61	32.0	.72	37.0	.81	42.0	.95	49.0
1.00 cm												
H_2O scattered	.04	2.1	.08	4.1	.10	5.2	.12	6.2	.13	6.7	.15	7.8
H_2O absorbed	.15	7.8	.17	8.8	.19	9.8	.20	10.4	.21	10.9	.22	11.4
Total lost	.35	18.0	.54	28.0	.68	35.0	.81	42.0	.90	47.0	1.05	54.0
2.00 cm												
H_2O scattered	.09	4.7	.13	6.7	.19	9.8	.21	10.9	.25	13.0	.28	14.5
H_2O absorbed	.18	9.3	.21	10.9	.23	11.9	.24	12.4	.24	12.4	.25	13.0
Total lost	0.43	22.0	0.63	33.0	0.81	42.0	0.94	49.0	1.05	54.0	1.21	63.0

These tables show that on the average about half the loss of energy in coming through the atmosphere is due to the scattering and absorption in the permanent gases of the atmosphere and half

[1] *Annals* of this Observatory, **2**, 112–113, 1908.

TABLE IV

WASHINGTON. ATMOSPHERIC ABSORPTION FOR DRY AIR AND DRY AIR PLUS DUST AND VARIOUS AMOUNTS OF WATER VAPOR

Altitude sea-level; Barometer 76.0 cm

Incident solar radiation, 1.93 15° C.-gram-calories per sq. cm per minute

AIR MASSES	$m=1$		$m=2$		$m=3$		$m=4$		$m=5$		$m=7$	
Precipitable Water Vapor	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost	Gram-Cal. Lost	Percentage Lost
0.00 cm												
Air scattered	0.18	9.3	0.33	17.1	0.44	22.8	0.53	27.5	0.61	31.6	0.73	37.8
Air absorbed	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.01	0.5	.01	0.5
Total lost	.19	10.0	.34	18.0	.45	23.0	.54	28.0	.62	32.0	.74	38.0
0.5 cm February 15												
H_2O scattered	.08	4.1	.15	7.8	.21	10.9	.26	13.5	.30	15.5	.37	19.2
H_2O absorbed	.12	6.2	.13	6.7	.14	7.3	.14	7.3	.14	7.3	.14	7.3
Total lost	.39	20.0	.62	32.0	.80	42.0	.94	49.0	1.06	55.0	1.25	65.0
1.8 cm October 4												
H_2O scattered	.26	13.5	.42	21.8	.53	27.5	.61	31.6	.67	34.7	.74	38.3
H_2O absorbed	.15	7.8	.16	8.3	.16	8.3	.14	7.3	.13	6.7	.11	5.7
Total	.60	31.0	.92	48.0	1.14	59.0	1.29	67.0	1.42	74.0	1.59	82.0
2.4 cm May 14												
H_2O scattered	.38	19.7	.56	29.0	.67	34.7	.74	38.3	.78	40.4	.83	43.0
H_2O absorbed	.16	8.3	.16	8.3	.15	7.8	.14	7.3	.13	6.7	.10	5.2
Total	0.73	38.0	1.06	55.0	1.27	66.0	1.42	74.0	1.53	79.0	1.67	86.0

is due to similar losses in the water vapor.[1] For the average amount of water vapor at Mount Wilson (0.7 cm precipitable water) the

[1] In considering these values of the relative absorption and scattering, a peculiarity of Bouguer's formula should be borne in mind. We have seen in any case that the total energy transmitted is $e_o(a_a a_w^w)^m$. However, if the air gets its share first, the amounts of energy transmitted and absorbed by the air and water vapor respectively are:

$$e_o a_a^m; \qquad e_o(1-a_a^m); \qquad e_o a_a^m \cdot a_w^{wm}; \qquad e_o a_a^m(1-a_w^{wm});$$

but if the water vapor gets its share first they are:

$$e_o(a_w^{wm} a_a^m); \qquad e_o a_w^{wm}(1-a_a^m); \qquad e_o a_w^{wm}; \qquad e_o(1-a_w^{wm}).$$

The final amounts transmitted are the same in each case, but the absorbent coming first gets the better chance, other things being equal. The values given in this section of the paper assume that the water vapor comes last, but since it extends to a very appreciable altitude, its share in the absorption relative to dry air is underestimated. The total effect of both together would be the same, irrespective of the distribution. These considerations in no way affect the use of the deflection in a band of water vapor to estimate the amount of water vapor or the absorptions as expressed in the next section, as in both these instances the use of ratios causes the inequalities just noted to appear with equal effect in numerator and denominator.

losses of solar energy due to dry air, the water vapor, and both together are on the average when the sun is in the zenith:

8 per cent (0.15 cal.); 9 per cent (0.17 cal.); 17 per cent (0.32 cal.).

When the sun is about 70° ($m = 2.9$) from the zenith, the corresponding values become:

20 per cent (0.39 cal.); 13 per cent (0.25 cal.); 33 per cent (0.64 cal.).

For Washington on the driest day (0.5 cm precipitable water) the corresponding values are:

10 per cent (0.19 cal.); 10 per cent (0.19 cal.); 20 per cent (0.38 cal.);
23 per cent (0.44 cal.); 19 per cent (0.37 cal.); 42 per cent (0.81 cal.).

(The loss due to the dust at Washington is included with that due to water vapor.) The far greater transparency of the air at Mount Whitney is largely due to the small amount of water vapor. On the days on which spectroscopic observations have been made there was never more than 0.1 cm precipitable water above the mountain. With this amount of water vapor the corresponding values are:

8 per cent (0.15 cal.); 4 per cent (0.08 cal.); 12 per cent (0.23 cal.);
17 per cent (0.33 cal.); 6 per cent (0.12 cal.); 23 per cent (0.45 cal.).

It seems perhaps strange at first sight that with increasing air masses the amount of absorption by water vapor (area of the bands) may decrease despite the increased amount of vapor in the path of beam, even seven fold in the extreme range in the table. This is principally due to the increasing air scattering (see last footnote) which leaves much less energy for the vapor to absorb. But this is continually helped by the decreasing efficiency of the increased amount of vapor as an absorber, as shown by the curve connecting the absorption with the amount of vapor (Fig. 3, *e*).

FRACTIONAL TRANSMISSION OF ENERGY BY WATER VAPOR

In general we cannot state what fraction of the radiation from any body whatever will be transmitted by a known amount of water vapor; for this depends upon the distribution of the energy

in the spectrum as well as upon the amount of vapor. In a perfectly pure spectrum coefficients of transmission might be determined for each wave-length which would probably hold as well in this case as for the more slowly varying molecular scattering. But this is not feasible. The next best scheme is to divide the spectrum into small regions, each containing one of the aqueous vapor bands, and to show by empirical curves how the transmission varies with the amount of vapor in these regions. The bands and limiting wave-lengths and deviations chosen are indicated in Fig. 2 by the broken lines at the bottom of the figure and are as shown in Table V.

TABLE V

Band	Range of Deviations	Range of Wave-Lengths
a	$92'.5-97'.5$	$0^{\mu}.70-0^{\mu}.74$
—	82.5 87.5	0.79 0.84
$\rho\sigma\tau$	70.0 80.0	0.86 0.90
Φ	55.0 67.5	1.03 1.23
Ψ	35.0 54.0	1.24 1.53
Ω	−7.5 35.0	1.53 2.19

Curves showing the fractional transmission in these regions as varying with the amount of water vapor are shown in Fig. 4. The nearer the distribution of energy in the spectrum is solar, the more accurately these values apply. Unfortunately the most powerful band, Ω, is very wide and lies in a part of the spectrum in which the distribution of energy varies widely as we pass from bodies of solar temperature through the more intense illuminants (Nernst lamp, e.g.) to terrestrial sources of radiation. In the case of the sun, 12 per cent of the total radiation sent to us lies in the region affected by this great band; in the case of the Nernst lamp about 20 per cent lies within this region, whereas for a source at 100° C. (373° K.) less than a tenth of 1 per cent lies within the whole region of wave-lengths less than 2 μ considered in this communication.

SUMMARY

The main object in view in the presentation of this communication has been to give definite answers to the two questions: How much is lost from the incoming solar energy in its transmission

through the different constituents of the atmosphere? What is the fractional transmission of energy by dry air and by aqueous vapor, and how does it vary as we pass from wave-length to wave-length through the spectrum?

The first question is answered in Tables II, III, and IV which give, for an elevation of 4420 m (Mount Whitney, barometer

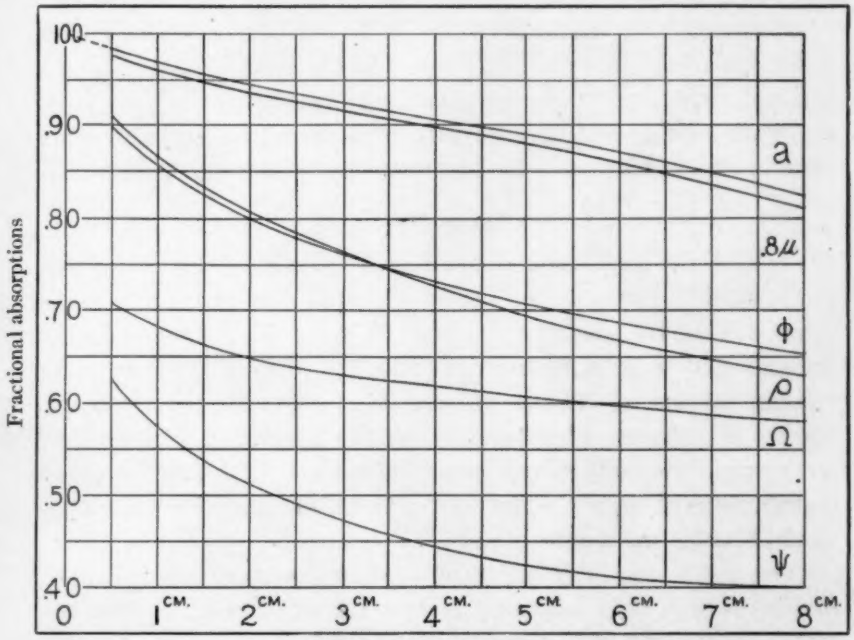

FIG. 4.—Fractional absorption of energy by water-vapor bands

44.7 cm), of 1730 m (Mount Wilson, barometer 62.3 cm), and sea-level (Washington), the amounts of heat as calories and percentages of the incident energy (1.93 15° C.-gram calories) scattered and absorbed at different air masses by dry air and by air containing various amounts of precipitable water.

Further, when we know the amount of precipitable water in the air and that the distribution of energy in the spectrum is approximately solar, by first reducing our observed curve to certain standard conditions we may determine through the curves of Fig. 3 what

areas of the energy-curve would be cut out by the water vapor and other absorption bands of atmospheric air.

The distribution of energy in the solar spectrum (outside the atmosphere) produced by a 60° ultra-violet glass prism is given in Table I.

The answer to the second problem, the fraction absorbed from wave-length to wave-length, will be found in Fig. 4 and as follows: The non-selective scattering due to dry air and associated with water vapor will be found in Table I in the lines indicated by $a_{a\lambda}$ and $a_{w\lambda}$, respectively. These values vary nearly continuously with the wave-length, and may be used with the e_o lines of the same table to find the intensity for any wave-length through the formula

$$e=e_o\{a_{a\lambda}\cdot a_{w\lambda}^{w}\}^{m},$$

where m is the air mass and equal to the secant of the zenith distance within 1 per cent for zenith distances less than 70°. This formula is for the altitude of Mount Wilson. For other altitudes the exponent m must be multiplied by the ratio of the barometer reading to that at Mount Wilson (62.3 cm). In the spectrum regions of selective absorption a further allowance is necessary, as shown in the body of this communication. Of course for other distributions of energy, other values would be used for e_o, and m would become the length of path.

For the bands of selective absorption the spectrum has been divided into certain indicated regions for which Fig. 4 gives the fractional transmission corresponding to definite amounts of water vapor.

It should be remarked that there is no reason to suspect that the selective (banded) absorption produced by a given amount of water in the form of vapor should be different, whether observed in the laboratory or in the atmosphere. However, in the case of non-selective scattering, the amount scattered by atmospheric vapor is greater than would be expected from the number of molecules of water vapor present; hence the use of the expression "associated with water vapor.". Liquid water scatters what would be expected from the number of molecules.[1]

[1] *Astrophysical Journal*, **38**, 392, 1913.

A comparison between observations of humidity from a balloon and nearly simultaneous spectroscopic determinations of the amounts of water vapor in the air shows an exceedingly satisfactory agreement.

This paper is the fifth of this series in this *Journal* discussing the transmission of radiation through moist and dry air and water vapor. The first (**35**, 149, 1912) furnished the laboratory calibration, with known amounts of water vapor, of the intensity of energy in certain absorption bands the depths of which could be very accurately measured bolometrically. The second (**37**, 359, 1913) gave some applications of the first in the spectroscopic determination of the water vapor above Mount Wilson and a comparison of these values with determinations by Hann's formula. The third (**38**, 392, 1913) treated of the non-selective scattering of dry air and water vapor for the spectrum region between the wave-lengths $0.35\ \mu$ and $2.00\ \mu$. The fifth gives the corresponding selective absorptions. The fourth (**40**, 435, 1914) was concerned with the application of the dry-air transmission coefficients to the determination of Avogadro's constant, the number of molecules in a gram-molecule of any gas.

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THE ELEMENTS OF THE ECLIPSING SYSTEMS TV, TW, TX CASSIOPEIAE AND T LEONIS MINORIS[1]

By R. J. McDIARMID

The following four eclipsing stars have been under observation by the writer for three successive seasons with the sliding-prism, polarizing photometer, attached to the 23-inch equatorial of the Princeton University Observatory. This paper gives the results obtained from the discussion of these systems.

TABLE I

Star	Position 1900 α	Position 1900 δ	Period in Days	Epoch of Primary Minimum J.D.G.H.M.T.
TV Cassiopeiae	$0^h13^m55^s$	58°35′	1.812635	2420117.742
TW Cassiopeiae	2 37 38	65 18.6	2.857293	2419823.432
TX Cassiopeiae	2 44 24	62 22.4	2.926870	2420448.923
T Leonis Minoris	9 42 33	33 45.2	3.0198965	2420573.698

Star	Spectrum	Magnitude			Eclipses* Duration		Discoverer
		Max.	Prim.	Sec.	Prim.	Sec.	
TV Cassiopeiae	B9	7.27	8.32	7.36	$6^h.24$ P	$6^h.24$ P	Astbury
TW Cassiopeiae	B9	8.29	8.91	8.86	7.54 A	6.94 T	Leavitt
TX Cassiopeiae	B3–B5	9.25	9.80	9.57	22.24 A	22.24 T	Leavitt
T Leonis Minoris	A0–A5	10.00	12.46	10.04	9.28 P	9.28 P	Leavitt

*P = partial. T = total. A = annular.

The periods of TV and TW Cassiopeiae have been determined from the writer's visual observations, while the periods of TX Cassiopeiae and T Leonis Minoris have been determined by combining the Princeton visual observations with Harvard photographic observations dating back to 1889. The epoch of minima for the four stars are all from the writer's visual observations. The spectral type of each star was determined by Miss Cannon and

[1] Dissertation presented to the faculty of Princeton University in candidacy for the degree of Doctor of Philosophy.

kindly communicated by Professor E. C. Pickering. The magnitudes were obtained by comparing the variable with stars in the same field, whose magnitudes have been determined photometrically at Harvard. These results are given in Table I.

The light-variations of the four systems have been carefully observed and are well defined by the respective light-curves. In all there have been over 35,000 measures of brightness made, sixteen measures constituting one complete observation. In discussing the observations a system of weights was adopted (maximum weight being 5), depending on condition of the sky, presence of dew or frost on object-glass, physical condition of the observer, and the accuracy of the recorded times. The observations were grouped into normals according to phase, each normal representing the weighted mean of five complete observations. The observations of the first three stars are well distributed throughout the entire period of variation. In the case of T Leonis Minoris, however, the observations are not so numerous and the normals do not all consist of five observations each.

TV CASSIOPEIAE

The following discussion is based on 620 observations of 16 settings made by comparing the variable with B.D.+58°29, magnitude 9.94 (the magnitude of B.D.+58°29 was obtained from the comparisons with TV Cassiopeiae, five separate determinations of whose magnitude at normal light are given in *Harvard Annals*, **45**). Each point in the curve, with one exception, represents the weighted mean of five observations, making 123 normals in all. This star has been observed by Astbury and Nijland. For Nijland's results, see Shapley's *Contribution No. 3, Princeton University Observatory*. Both observers give the variation at primary minimum as one magnitude; no mention is made, however, of a secondary eclipse. It was partly for that reason, on Dr. Shapley's suggestion, that an extensive study of the system was decided on. The Princeton observations indicate a period 5.7 seconds longer than that given by Astbury; it was found that the primary eclipse was nearly $1^{\mathrm{M}}.05$ in depth and that there was a secondary eclipse of $0^{\mathrm{M}}.09$. The secondary minimum is slightly displaced,

coming 17 minutes before mid-period, showing that the orbit is eccentric.

The light between eclipses does not remain constant, showing the presence of reflection and ellipticity. These two effects were removed from the light-curve in much the same manner as that recorded in Russell and Shapley's paper on Z Draconis.[1] From a preliminary discussion, values $a=1.003$, $b=0.040$, $c=0.045$ were obtained. Corrections to these quantities were made from a least-squares solution, taking into account depth of secondary minimum, as well as the time of secondary. Assumed depth of secondary $d=0.104$ and time of secondary $=21^h24^m$.

The conditional equation is of the form:

$$l=a-b\cos\theta-c\cos^2\theta-nd+e.$$

The constants in the equation have the same signification as in the foregoing reference. Table III contains the conditional equations used in the least-squares solution.

TABLE II

TABLE OF OBSERVATIONS. TV CASSIOPEIAE

PRIMARY MINIMUM

Normal No.	Phase	Mag. Diff. $(v-a)$	O.−C.$_u$	O.−C.$_d$	Normal No.	Phase	Mag. Diff. $(v-a)$	O.−C.$_u$	O.−C.$_d$
1..	$-4^h54^m.1$	$2^M.641$	$+0^M.025$	$+0^M.025$	20..	$-0^h06^m.2$	$1^M.635$	$0^M.000$	$+0^M.003$
2..	4 19.7	2.622	+0.012	+0.012	21..	+0 10.7	1.625	−0.015	−0.010
3..	3 46.5	2.577	−0.010	−0.008	22..	0 25.6	1.671	−0.008	0.000
4..	3 34.8	2.578	+0.008	+0.010	23..	0 40.4	1.744	−0.010	0.000
5..	3 25.9	2.497	−0.052	−0.054	24..	0 51.3	1.799	−0.015	−0.006
6..	3 14.6	2.512	−0.012	−0.015	25..	1 02.3	1.891	+0.006	+0.006
7..	3 04.6	2.482	−0.013	−0.020	26..	1 11.9	1.949	0.000	0.000
8..	2 49.6	2.438	−0.012	−0.015	27..	1 23.2	1.999	−0.017	−0.020
9..	2 31.6	2.363	−0.010	−0.010	28..	1 36.1	2.036	−0.060	−0.060
10..	2 16.7	2.288	−0.014	−0.016	29..	1 48.3	2.132	−0.038	−0.035
11..	2 02.5	2.211	−0.020	−0.025	30..	1 59.0	2.188	−0.030	−0.038
12..	1 43.4	2.163	+0.012	+0.012	31..	2 09.0	2.244	−0.034	−0.034
13..	1 26.7	2.078	+0.032	+0.032	32..	2 19.8	2.324	+0.005	+0.003
14..	1 11.3	1.955	+0.013	+0.020	33..	2 32.9	2.385	−0.004	−0.006
15..	0 58.5	1.828	−0.020	−0.010	34..	2 43.0	2.437	+0.008	+0.005
16..	0 47.0	1.782	−0.003	0.000	35..	2 59.5	2.535	+0.045	+0.040
17..	0 35.2	1.738	+0.010	+0.015	36..	3 19.3	2.606	+0.065	+0.065
18..	0 27.7	1.692	+0.005	+0.010	37..	3 54.9	2.646	+0.042	+0.052
19..	0 17.0	1.682	+0.020	+0.023	38..	4 52.5	2.634	+0.015	+0.015

[1] *Astrophysical Journal*, 39, 405, 1914.

TABLE II—*Continued*

CONSTANT LIGHT AND SECONDARY MINIMUM

Normal No.	Phase	Mag. Diff. (v−a)	O.−C.	Normal No.	Phase	Mag. Diff. (v−a)	O.−C.
39......	$5^h55^m.5$	$2^M.665$	$+0^M.030$	83....	$22^h25^m.2$	$2^M.611$	$+0^M.022$
40......	6 27.8	2.666	+0.028	84....	22 36.6	2.585	−0.006
41......	6 50.8	2.653	+0.014	85....	22 49.9	2.550	−0.043
42......	7 22.3	2.603	−0.033	86....	23 01.4	2.633	+0.033
43......	7 40.3	2.710	+0.056	87....	23 14.8	2.615	+0.010
44......	7 53.3	2.653	+0.005				
				88....	23 29.0	2.589	−0.020
45......	8 17.8	2.639	−0.013	89....	23 44.5	2.637	+0.015
46......	8 56.3	2.668	+0.010	90....	24 06.9	2.650	+0.020
47......	9 15.5	2.615	−0.044	91....	24 29.5	2.653	+0.012
48......	10 12.0	2.688	+0.016	92....	24 51.6	2.661	+0.012
49......	11 16.9	2.674	+0.004	93....	25 15.4	2.641	−0.015
50......	12 09.7	2.713	+0.038	94....	25 40.1	2.674	+0.006
51......	12 49.2	2.689	+0.006	95....	25 15.3	2.644	−0.022
52......	13 10.1	2.696	+0.014	96....	25 49.2	2.652	−0.017
53......	13 26.7	2.660	−0.020				
				97....	27 18.4	2.676	+0.004
54......	13 42.9	2.658	−0.022	98....	27 54.9	2.650	−0.023
55......	13 59.1	2.708	+0.027	99....	28 34.4	2.689	+0.010
56......	14 21.3	2.709	+0.027	100....	29 03.1	2.692	+0.012
57......	14 51.7	2.647	−0.024	101....	29 22.5	2.688	+0.008
58......	16 35.5	2.648	−0.016	102....	29 44.9	2.703	+0.023
59......	16 55.7	2.676	+0.010	103....	30 13.9	2.676	−0.004
60......	17 15.5	2.668	+0.006	104....	30 43.6	2.685	+0.005
61......	17 38.9	2.616	−0.040	105....	31 33.6	2.703	+0.023
62......	18 17.4	2.643	+0.005	106....	32 07.0	2.692	+0.018
63......	18 56.6	2.586	−0.045				
				107....	32 26.4	2.679	+0.004
64......	19 25.5	2.628	+0.017	108....	32 53.6	2.670	+0.003
65......	19 46.7	2.640	+0.040	109....	33 20.4	2.659	+0.002
66......	19 58.3	2.602	+0.007	110....	33 47.8	2.618	−0.013
67......	20 09.1	2.594	+0.003	111....	34 25.1	2.623	−0.044
68......	20 20.4	2.552	−0.003	112....	34 54.3	2.638	−0.012
69......	20 35.2	2.632	+0.050	113....	35 19.7	2.677	+0.028
70......	20 46.4	2.635	+0.060	114....	35 50.4	2.622	−0.016
71......	20 56.1	2.603	+0.030	115....	36 20.2	2.658	+0.022
72......	21 02.2	2.599	+0.026	116....	36 42.0	2.631	+0.002
73......	21 09.1	2.562	−0.003				
				117....	36 55.4	2.592	−0.036
74......	21 15.4	2.550	−0.010	118....	37 04.8	2.598	−0.030
75......	21 21.7	2.525	−0.030	119....	37 20.8	2.581	−0.040
76......	21 28.1	2.531	−0.022	120....	37 33.5	2.644	+0.022
77......	21 35.6	2.555	−0.004	121....	37 47.5	2.608	−0.010
78......	21 42.4	2.534	−0.022	122....	38 02.9	2.603	−0.016
79......	21 49.7	2.570	+0.005	123....	38 16.1	2.599	−0.007
80......	21 58.4	2.602	+0.030				
81......	22 04.8	2.576	+0.004				
82......	22 14.5	2.565	−0.014				

TABLE III

CONDITIONAL EQUATIONS OF TV CASSIOPEIAE

δa	δb	δc	δd	e	O.−C.
1.0	−0.64	−0.41	....	....	−0.026
1.0	−0.43	−0.18	....	....	+0.005
1.0	−0.14	−0.02	....	....	−0.004
1.0	+0.29	−0.08	....	....	+0.006
1.0	+0.47	−0.22	....	....	−0.002
0.7	+0.52	−0.40	−0.07	+0.01	−0.009
0.9	+0.77	−0.67	−0.45	+0.11	−0.019
0.9	+0.86	−0.83	−0.72	+0.10	+0.004
1.0	+0.98	−0.97	−0.80	+0.06	+0.009
0.9	+0.89	−0.89	−0.89	+0.04	+0.026
0.9	+0.90	−0.90	−0.86	−0.10	−0.012
0.9	+0.90	−0.90	−0.74	−0.10	−0.001
0.9	+0.89	−0.89	−0.63	−0.08	+0.007
0.9	+0.88	−0.87	−0.48	−0.08	−0.004
0.9	+0.86	−0.83	−0.27	−0.05	+0.005
1.0	+0.91	−0.74	−0.00	−0.01	+0.009
1.0	+0.79	−0.56	....	....	−0.021
1.0	+0.60	−0.36	....	....	−0.005
1.0	+0.37	−0.13	....	....	+0.019
1.0	+0.05	−0.00	....	....	+0.010
1.0	−0.21	−0.04	....	....	−0.025
1.0	−0.47	−0.22	....	....	+0.005
1.0	−0.62	−0.38	....	....	−0.020
1.0	−0.72	−0.51	....	....	−0.002

The following normal equations were formed:

$$
\begin{array}{llllll}
+22.900\delta a & +\ 7.849\delta b & -11.201\delta c & -5.468\delta d & -0.086e & =-0.0135 \\
+\ 7.849 & +11.365 & -\ 7.528 & -5.262 & -0.083 & =+0.0061 \\
-11.201 & -\ 7.528 & +\ 8.708 & +5.217 & +0.073 & =-0.0089 \\
-\ 5.468 & -\ 5.262 & +\ 5.217 & +4.142 & +0.060 & =-0.0237 \\
-\ 0.086 & -\ 0.083 & +\ 0.073 & +0.060 & +0.063 & =+0.0013
\end{array}
$$

The adopted values with their probable errors are:

$a=1.002\pm0.003$ $\qquad$ $d=0.080\pm0.009$

$b=0.037\pm0.004$ $\qquad$ $e=21^h28^m\pm7^m$

$c=0.052\pm0.009$

Probable error of one normal place outside principal minimum = ± 0.016.

The observations were now "rectified" by use of the formula

$$l \text{ rectified} = \frac{l \text{ observed} + b(1+\cos\theta)}{(1+b)(a-c\cos^2\theta)}$$

after which the method of solution for spherical stars equally bright on both sides may be applied with slight modification (see *Astrophysical Journal;* **36**, 406, 1912).

When the star-disks are assumed to be of uniform brightness, the value of the function $\chi(k, \alpha_0, \frac{1}{4})$ which defines the form of the light-curve for the principal minimum was found to be 1.876. With the aid of Table III[1] and this value and the depth of the rectified primary and secondary minima, the ratio of the radii of the stars, k, comes out 0.95 and the percentage of obscuration, a_0, 0.628. The other elements were then determined as indicated in *Astrophysical Journal*, **36**, 406, 1912.

For disks darkened at the limb, the observations are represented equally well. From the "rectified" depth of primary and secondary eclipses and the equation $Q(k, \alpha_0) = \dfrac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}$ values of k and α_0 were computed; these values for k range from $k=0.82$ to 1.00 and from $\alpha_0 = 0.62$ to 0.64. Upon computing $\chi(k, \alpha_0, 0)$, $\chi(k, \alpha_0, \frac{1}{4})$, and $\chi(k, \alpha_0, \frac{3}{4})$ from the observed curve, it was found that the possible range of values for k and α_0 was very small. The most satisfactory curve was found for $\alpha_0 = 0.62$ and $k=0.884$. The resulting light-curve was computed by means of Tables IIIx and IIx.

The elements for the two solutions are given in the table of results. The residuals for the uniform and darkened solutions are scaled from the computed curves and are given in the table of observations. The probable error of one normal place in the principal eclipse is $\pm 0^{M}.017$ for uniform solution and $\pm 0^{M}.017$ for the darkened. Fig. 1 gives the theoretical light-curve derived from the uniform solution, and Fig. 2 diagrams of the systems resulting from the uniform and the darkened solutions.

TW CASSIOPEIAE

This star was discovered by Miss Leavitt on the Harvard plates, and has been observed by Münch[2] and Zinner.[3] From the pub-

[1] This and following references are to tables in *Astrophysical Journal*, **35**, **36**, 1912.

[2] *Astronomische Nachrichten*, **182**, 113, 1909.

[3] *Ibid.*, **190**, 377, 1912; **195**, 453, 1913.

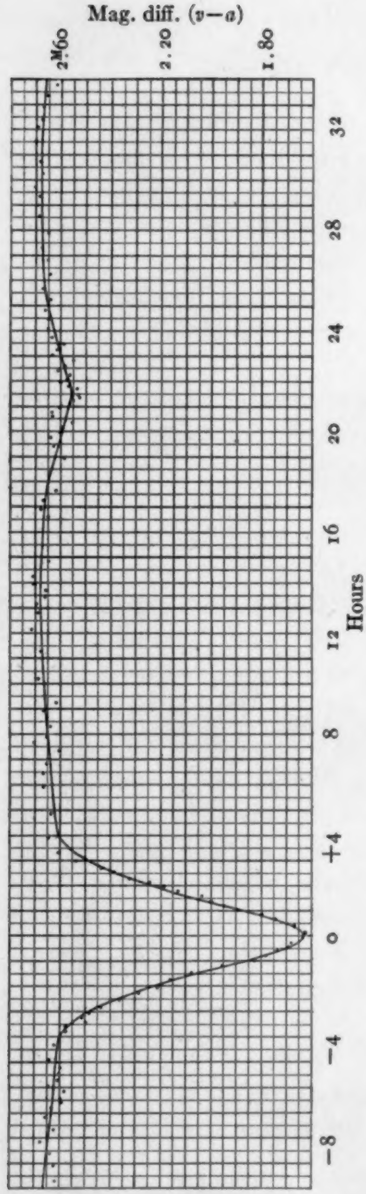

FIG. 1.—Mean light-curve of TV Cassiopeiae

lished note of Münch, the star was given as an Algol variable with a period of 10 days, or probably irregular. The range of his variation was $8^{M}.0$ to $8^{M}49$. Zinner also thought it irregular from early observations but later found it to be a variable of the Algol type with a period of $1^{d}10^{h}16^{m}.7$ and a variation from $8^{M}.3$ to $9^{M}.0$.

The variation of this star is well defined by the Princeton observations. In all, 820 complete observations have been made on the star by comparing it with B.D. 64°343, magnitude 10.49 (the magnitude of which was obtained by photometric comparison with three stars whose magnitudes are given in *Harvard Annals*, **64**). The observations were grouped in the usual manner into 172 normals. The eclipses have been observed five times each, in order to define the curve precisely. From the plot of the observations it appeared that the alternate eclipses, which had been considered similar, were of slightly different depth. The assembling of the observations on a period double Zinner's and the plotting of the normals confirmed this suspicion. The period is undoubtedly double that given by Zinner and is confirmed by the following

observed phenomena in the light-curve: first, there is a difference in depth of the two eclipses of $0^{M}.05$; second, the interval from primary eclipse to secondary is 7.8 minutes longer than from secondary to the following primary; third, the two eclipses are of different duration, the primary being at least 36 minutes longer than the secondary. The last two facts show that the orbit is eccentric and in this case, as will be shown later, both the eccentricity and longitude of periastron can be determined from the light-curve. The range of variation agrees closely with Zinner's determination, as the loss of light is $0^{M}.62$ for the primary and $0^{M}.57$ for the secondary.

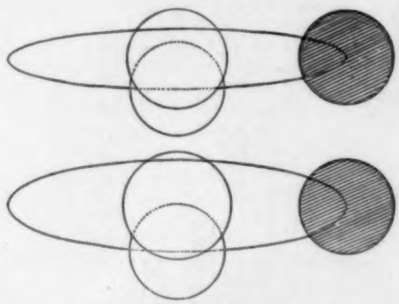

FIG. 2.—Diagrams for uniform and darkened solutions for TV Cassiopeiae.

The light between eclipses remains sensibly constant. In this system we have two well-defined eclipses of nearly the same depths and duration. The orbital eccentricity considerably complicates the solution, and several trials were made before reaching a satisfactory representation of the light-curve, in which, on the hypothesis of uniformly bright star-disks, the principal eclipse is annular and the secondary total. The value of k was 0.858, obtained from the relation $1-\lambda p+\frac{1-\lambda s}{k}=1$, where $1-\lambda p=0.436$ and $1-\lambda s=0.408$, the loss of light at primary and secondary eclipse respectively. From the Table II of the function $\psi(ka_0)$ for uniform disks, we can compute the light-curve for each minimum separately, adjusting the quantities A and B to get the best representation of the observations. For primary eclipse $A=0.0176$, $B=0.01430$, and for secondary, $A=0.01516$ and $B=0.00286$. Following the notation as indicated in *Astrophysical Journal*, **36**, 406, 1912, the radii of the stars from the two minima came out $r=0.183$, $r=0.157$ for the primary, and $r=0.169$ and $r=0.145$ for the secondary. It was also found that the semi-durations of the two eclipses differed by 18 minutes, the primary being the longer. The difference in

TABLE IV

Table of Observations. TW Cassiopeiae

PRIMARY MINIMUM

Normal No.	Phase	Mag. Diff. $(a-v)$	$O.-C._u$	$O.-C._d$	Normal No.	Phase	Mag. Diff. $(a-v)$	$O.-C._u$	$O.-C._d$
1..	$-0^d4^h44^m.5$	$2^M.101$	$+0^M.001$	$+0^M.001$	18..	$+0^d0^h01^m.8$	$1^M.497$	$+0^M.017$	$+0^M.025$
2..	3 59.8	2.105	+0.005	0.000	19..	0 11.0	1.478	0.000	−0.005
3..	3 39.3	2.074	−0.020	−0.015	20..	0 21.9	1.506	0.000	0.000
4..	3 09.4	2.019	−0.038	−0.040	21..	0 32.8	1.502	−0.032	−0.028
5..	2 41.6	1.998	0.000	−0.003	22..	0 43.8	1.569	−0.010	−0.010
6..	2 15.0	1.842	−0.036	−0.038	23..	0 55.3	1.615	−0.012	−0.015
7..	1 54.9	1.856	−0.010	−0.015	24..	1 05.4	1.668	−0.010	−0.008
8..	1 42.5	1.830	0.000	0.000	25..	1 17.9	1.702	−0.025	−0.027
9..	1 31.1	1.829	+0.025	+0.022	26..	1 30.6	1.790	0.000	0.000
10..	1 20.9	1.773	+0.025	+0.020	27..	1 41.0	1.833	+0.008	+0.006
11..	1 11.5	1.701	0.000	0.000	28..	1 53.3	1.889	+0.016	+0.013
12..	0 57.5	1.605	−0.030	−0.030	29..	2 05.5	1.903	−0.003	−0.010
13..	0 43.1	1.604	+0.025	+0.025	30..	2 26.9	1.968	0.000	−0.003
14..	0 33.7	1.510	−0.017	−0.018	31..	3 05.3	2.053	+0.004	0.000
15..	0 25.8	1.492	−0.010	−0.010	32..	3 45.9	2.113	+0.013	+0.013
16..	0 16.7	1.472	−0.003	−0.010	33..	4 09.7	2.072	−0.028	−0.028
17..	0 07.9	1.464	−0.012	−0.006	34..	4 48.2	2.105	+0.005	+0.050

SECONDARY MINIMUM

Normal No.	Phase	Mag. Diff. $(a-v)$	$O.-C._u$	$O.-C._d$	Normal No.	Phase	Mag. Diff. $(a-v)$	$O.-C._u$	$O.-C._d$
1..	$1^d6^h01^m.8$	$2^M.102$	$+0^M.002$	$+0^M.002$	20..	$1^d10^h34^m.9$	$1^M.504$	$-0^M.024$	$-0^M.012$
2..	6 36.8	2.068	−0.032	−0.032	21..	10 42.3	1.539	+0.003	+0.015
3..	6 54.0	2.108	+0.008	+0.008	22..	10 50.8	1.594	+0.025	+0.040
4..	7 21.1	2.069	−0.010	−0.020	23..	10 59.3	1.605	0.000	−0.008
5..	7 43.4	2.022	−0.010	−0.030	24..	11 06.7	1.612	−0.010	0.000
6..	8 02.7	2.060	+0.070	+0.045	25..	11 12.2	1.607	−0.050	−0.035
7..	8 16.1	2.042	+0.050	+0.027	26..	11 20.5	1.667	−0.016	−0.006
8..	8 33.0	1.925	+0.030	+0.008	27..	11 27.7	1.723	+0.003	+0.002
9..	8 49.2	1.830	−0.010	−0.030	28..	11 37.2	1.745	−0.010	−0.012
10..	9 04.0	1.816	+0.022	+0.018	29..	11 51.7	1.843	+0.023	+0.015
11..	9 16.6	1.753	+0.006	+0.013	30..	12 07.6	1.856	−0.010	−0.020
12..	9 28.7	1.696	−0.005	0.000	31..	12 25.1	1.946	+0.014	−0.005
13..	9 39.6	1.609	−0.040	−0.012	32..	12 40.4	2.003	+0.026	−0.005
14..	9 48.4	1.571	−0.032	−0.010	33..	12 59.9	2.070	+0.050	+0.020
15..	9 57.0	1.541	−0.020	−0.008	34..	13 23.2	2.072	+0.003	+0.006
16..	10 03.9	1.519	−0.020	−0.012	35..	13 51.2	2.093	−0.006	−0.007
17..	10 10.1	1.561	+0.028	+0.036	36..	14 17.8	2.083	−0.015	−0.017
18..	10 18.7	1.528	−0.003	+0.007	37..	14 40.5	2.092	−0.008	−0.008
19..	10 27.3	1.552	+0.020	+0.036	38..	15 11.4	2.096	−0.004	−0.004

TABLE IV—*Continued*

CONSTANT-LIGHT OBSERVATIONS. TW CASSIOPEIAE

Normal No.	Phase	Mag. Diff. $(a-v)$	O.−C.	Normal No.	Phase	Mag. Diff. $(a-v)$	O.−C.
1......	$0^d\ 5^h 14^m.0$	$2^M.099$	$-0^M.001$	44.....	$1^d\ 2^h 49^m.2$	$2^M.069$	$-0^M.031$
2......	5 41.2	2.112	+0.012	45.....	3 08.8	2.082	−0.018
3......	6 10.4	2.095	−0.005	46.....	3 28.0	2.117	+0.017
4......	6 35.3	2.116	+0.016	47.....	3 41.5	2.145	+0.045
5......	6 54.4	2.076	−0.024	48.....	3 53.0	2.106	+0.006
6......	7 20.5	2.113	+0.013	49.....	4 16.5	2.089	−0.011
7......	7 40.5	2.065	−0.035	50.....	4 43.7	2.090	−0.010
8......	8 05.3	2.115	+0.015	51.....	4 58.4	2.102	+0.002
9......	8 43.2	2.109	+0.009	52.....	5 12.5	2.105	+0.005
10......	9 35.4	2.111	+0.011	53.....	5 26.6	2.108	+0.008
11......	10 14.4	2.069	−0.031	54.....	5 38.6	2.103	+0.003
12......	10 35.3	2.112	+0.012	55.....	15 11.4	2.096	−0.004
13......	10 59.4	2.118	+0.018	56.....	15 35.0	2.100	0.000
14......	11 28.1	2.074	−0.026	57.....	16 25.6	2.102	+0.002
15......	11 35.4	2.071	−0.029	58.....	16 44.9	2.101	+0.001
16......	12 26.9	2.084	−0.016	59.....	17 08.4	2.101	+0.001
17......	13 28.6	2.083	−0.017	60.....	17 45.7	2.066	−0.034
18......	14 06.4	2.113	+0.013	61.....	18 49.8	2.085	−0.015
19......	14 26.5	2.112	+0.012	62.....	19 13.8	2.105	+0.005
20......	14 47.2	2.097	−0.003	63.....	19 41.2	2.110	+0.010
21......	15 29.6	2.092	−0.008	64.....	20 20.7	2.086	−0.014
22......	15 57.3	2.081	−0.019	65.....	21 06.7	2.124	+0.024
23......	16 19.8	2.105	+0.005	66.....	21 27.9	2.126	+0.026
24......	16 49.4	2.095	−0.005	67.....	21 54.1	2.078	−0.032
25......	17 11.9	2.078	−0.022	68.....	22 28.8	2.112	+0.012
26......	17 37.8	2.143	+0.043	69.....	23 08.8	2.116	+0.016
27......	18 12.2	2.093	−0.007	70.....	23 48.9	2.096	−0.004
28......	19 00.0	2.082	−0.018	71.....	2 0 36.3	2.106	+0.006
29......	19 24.9	2.133	+0.033	72.....	1 28.8	2.118	+0.018
30......	19 50.2	2.108	+0.008	73.....	2 16.1	2.087	−0.013
31......	20 07.5	2.078	−0.022	74.....	3 10.2	2.109	+0.009
32......	20 51.8	2.102	+0.002	75.....	4 05.8	2.096	−0.004
33......	20 55.0	2.074	−0.026	76.....	5 08.6	2.103	+0.003
34......	21 22.6	2.092	−0.008	77.....	5 48.7	2.058	−0.042
35......	21 48.0	2.112	+0.012	78.....	6 19.0	2.080	−0.020
36......	22 02.6	2.102	+0.002	79.....	6 50.7	2.131	+0.031
37......	22 14.4	2.124	+0.024	80.....	7 26.0	2.098	+0.002
38......	22 29.1	2.090	−0.010	81.....	8 13.1	2.107	+0.007
39......	22 51.1	2.102	+0.002	82.....	8 50.2	2.099	−0.001
40......	23 30.7	2.108	+0.008	83.....	9 26.9	2.091	−0.009
41......	1 1 09.3	2.079	−0.031	84.....	9 53.4	2.124	+0.024
42......	2 09.2	2.122	+0.022	85.....	10 10.6	2.120	+0.020
43......	2 32.3	2.110	+0.010	86.....	10 30.9	2.061	−0.039

TABLE IV—*Continued*

Normal No.	Phase	Mag. Diff. ($a-v$)	O.–C.	Normal No.	Phase	Mag. Diff. ($a-v$)	O.–C.
87......	10^h50^m3	2.093	−0.007	94.....	12^h19^m6	2.113	+0.013
88......	11 08.5	2.100	0.000	95.....	12 36.0	2.087	−0.013
89......	11 17.9	2.108	+0.008	96.....	13 26.3	2.121	+0.021
90......	11 30.9	2.111	+0.011	97.....	13 53.0	2.108	+0.008
91......	11 42.9	2.106	+0.006	98.....	14 12.3	2.107	+0.007
92......	11 55.5	2.069	−0.031	99.....	15 06.6	2.115	+0.015
93......	12 07.2	2.112	+0.012				

the computed radii of the stars, as well as in the duration of the eclipses, is due to the orbit being eccentric.

By means of the equations (30) of Professor Russell's paper[1]

Primary Minimum	Secondary Minimum
$r_1'=r_1(1-g\eta)$	$r_1''=r_1(1+g\eta)$
$\cot i_1'=\cot i(1-2g\eta)$	$\cot i''=\cot i(1+2g\eta)$

$e \sin \omega$ was determined from the two solutions. The values were in good agreement, being -0.0388 from the values of r, and -0.0418 from those of $\cot i$. The remarkable agreement of the two separate determinations of $e \sin \omega$ from the light-curve is a strong confirmation of the eclipse theory. The quantity $e \cos \omega$ was determined from the displacement of the secondary minimum

$$e \cos \omega = \frac{\pi(t_2 - t_1 - \frac{1}{2}P)}{P(\operatorname{cosec}^2 i + 1)}$$

by the use of the equations (32) of the same paper, whence e and ω were readily determined. The remaining elements of the system were computed in the ordinary way.

In the solution for stars darkened toward the edge, it was found that the observations could not be represented with an annular and total eclipse. The eclipses were therefore considered partial and were drawn slightly deeper, $1-\lambda p=0.440$ and $1-\lambda s=0.420$. The eccentricity resulting from the uniform solution was adopted in the preliminary results. It was found that the observations were best represented for the stars equal. The values for k of 0.90 and 0.95 were tried, however, but the results were not satisfactory. By means of the equations $\frac{1-\lambda p}{\alpha_1}+\frac{1-\lambda s}{\alpha k^2}=1$, and $g=\frac{(p_2-p_{1})k}{2+(p_2+p_1)k}$,

[1] *Astrophysical Journal*, **36**, 57, 1912.

where $1-\lambda p$, $1-\lambda s$, k and g are known, values of α_1 and α_2 were found. The curve for each minimum was then computed separately. It was found that a slight increase in the eccentricity would improve the agreement. The eccentricity was finally adopted as 0.05, and curves were computed for each minimum which satisfy the observations remarkably well.

The elements obtained from the solution are given in the table of results. The residuals for both uniform and darkened solutions have been scaled off from the computed curves for primary and secondary minima and are given in the table of observations. The probable error of one normal place on the uniform hypothesis is ± 0.015, and for the darkened ± 0.013.

Fig. 3 gives the computed curve for stellar disks of uniform brightness, and Fig. 4 diagrams of the stars at elongation and at primary eclipse.

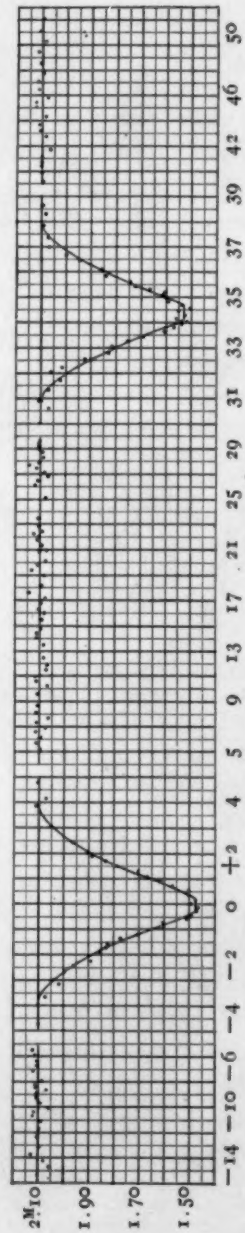

FIG. 3.—Mean light-curve of TW Cassiopeiae

TX CASSIOPEIAE

This variable was discovered by Miss Leavitt[1] on the Harvard plates and announced as of the Algol type with a variation from $8^{\mathrm{M}}.8$ to $9^{\mathrm{M}}.4$. It has been observed by Zinner[2] and was first noted by him as an irregular variable; later he found it to be eclipsing, and gave its period as $2^{d}22^{h}13^{m}.9$. Zinner notes that there are two minima, the secondary having a flat bottom; eclipses last sixteen hours and the variation is $9^{\mathrm{M}}.4$ to $10^{\mathrm{M}}.0$.

[1] *Harvard Circular*, No. 127.

[2] *Astronomische Nachrichten*, **195**, 453, 1913.

In referring the published Harvard photographic observations to these elements, Zinner found they were not satisfied; the interval between minima seemed to be of variable duration. He concluded that the case was similar to Y Cygni, with the line of apsides in motion.

This star has offered considerable difficulty in the photometric study. The period, owing to the nature of the light-variation, for a long time was doubtful. The small range of $0^{\mathrm{M}}.55$ for primary and $0^{\mathrm{M}}.32$ for secondary, together with the long duration of eclipses (over twenty-one hours), made the observing of a complete minimum at any one time impossible, even during the long nights of winter. Through the kindness of Professor E. C. Pickering in sending me a long series of Harvard photographic observations of this star dating as far back as 1889, I have been able, by combining these with the visual observations, to establish the period very accurately. The observations over this interval of twenty-five years present no evidence, as suggested by Zinner, of a motion of the line of apsides. In fact, it was found that the two plates referred to are not epochs of faintness, but of brightness. The star was discovered on these plates by comparison with one where the star was faint. After a large number of observations on the system had been made, it was found that the comparison star used was also a variable of range about $0^{\mathrm{M}}.2$; its light being nearly constant about half the time with increases of two or three hours' duration at apparently irregular intervals of from four to eight hours or more. A detailed discussion of this star and the methods adopted for correcting the observations will appear in a separate paper to be published in the *Journal of the R.A.S.* of Canada.

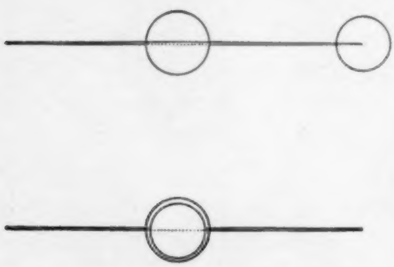

FIG. 4.—Elongation and primary eclipse of TW Cassiopeiae.

The light-variation of TX Cassiopeiae is very well defined by the Princeton observations. In all, 585 complete observations were

TABLE V

TABLE OF OBSERVATIONS. TX CASSIOPEIAE

Normal No.	Phase	Mag. Diff. (v−a)	O.−C.$_d$	Normal No.	Phase	Mag. Diff. (v−a)	O.−C.$_d$
1....	$-0^d15^h02^m0$	0^M334	$+0^M005$	46...	$0^d15^h53^m1$	0^M347	-0^M010
2....	11 59.8	0.383	−0.022	47...	16 15.1	0.351	−0.016
3....	11 39.1	0.357	+0.005	48...	18 17.5	0.339	−0.010
4....	10 50.0	0.392	−0.012	49...	18 57.9	0.313	+0.015
5....	9 05.4	0.429	−0.010	50...	20 49.7	0.333	0.000
6....	8 34.8	0.411	+0.020	51...	21 20.3	0.320	+0.018
7....	7 30.3	0.474	0.000	52...	22 16.0	0.345	−0.005
8....	6 27.8	0.513	+0.012	53...	23 05.4	0.362	−0.013
9....	4 54.2	0.643	−0.010	54...	23 30.2	0.345	+0.010
10....	4 21.7	0.682	−0.003	55...	23 53.6	0.399	−0.038
11....	4 01.2	0.737	−0.030	56...	1 00 12.8	0.353	+0.008
12....	3 33.2	0.742	+0.002	57...	0 37.3	0.364	+0.005
13....	2 55.1	0.796	−0.004	58...	1 11.3	0.384	−0.005
14....	2 24.6	0.786	+0.034	59...	1 31.6	0.396	−0.012
15....	1 47.3	0.874	−0.020	60...	1 47.0	0.409	−0.016
16....	1 12.3	0.891	−0.018	61...	2 14.0	0.386	+0.014
17....	0 54.2	0.891	−0.010	62...	2 43.1	0.443	−0.030
18....	−0 0 16.4	0.882	+0.005	63...	3 35.2	0.435	+0.012
19....	+0 0 01.4	0.900	−0.005	64...	4 47.3	0.526	−0.015
20....	0 21.2	0.879	+0.010	65...	5 40.8	0.558	−0.010
21....	0 50.5	0.883	+0.004	66...	6 00.6	0.557	+0.008
22....	1 11.5	0.858	+0.012	67...	6 28.8	0.621	−0.032
23....	1 40.6	0.900	−0.044	68...	7 05.2	0.607	+0.008
24....	2 15.1	0.839	−0.008	69...	7 29.5	0.627	+0.006
25....	2 40.9	0.828	−0.022	70...	7 47.1	0.636	+0.004
26....	2 55.6	0.782	+0.008	71...	8 09.0	0.666	−0.018
27....	3 19.2	0.770	−0.006	72...	8 26.0	0.665	−0.015
28....	3 36.2	0.742	0.000	73...	8 42.0	0.648	+0.005
29....	3 58.3	0.749	−0.050	74...	9 00.2	0.668	−0.008
30....	4 15.3	0.653	+0.030	75...	9 27.2	0.662	0.000
31....	6 22.3	0.498	+0.033	76...	10 09.5	0.665	+0.003
32....	7 02.2	0.503	−0.014	77...	10 48.4	0.660	+0.006
33....	7 58.4	0.441	+0.012	78...	11 40.8	0.674	−0.008
34....	8 26.6	0.427	+0.010	79...	12 17.0	0.667	−0.004
35....	9 20.8	0.398	+0.012	80...	12 42.5	0.660	+0.003
36....	10 46.7	0.366	+0.014	81...	13 01.8	0.696	−0.045
37....	11 39.8	0.395	−0.025	82...	13 20.2	0.660	−0.004
38....	12 05.0	0.351	+0.008	83...	13 37.2	0.632	+0.020
39....	12 28.7	0.391	−0.035	84...	13 56.5	0.647	−0.005
40....	12 51.0	0.353	0.000	85...	14 48.3	0.606	+0.014
41....	13 16.5	0.313	+0.035	86...	15 39.7	0.588	−0.008
42....	13 36.5	0.366	−0.020	87...	16 12.2	0.542	+0.010
43....	14 15.3	0.344	0.000	88...	16 49.6	0.539	−0.018
44....	15 09.4	0.307	+0.033	89...	17 19.7	0.500	−0.008
45....	15 36.1	0.318	+0.018	90...	17 42.5	0.454	+0.020

TABLE V—*Continued*

Normal No.	Phase	Mag. Diff. ($v-a$)	O.–C.$_d$	Normal No.	Phase	Mag. Diff. ($v-a$)	O.–C.$_d$
91....	$0^d19^h54^m1$	$0^M.393$	$+0^M.005$	99...	4^h22^m3	$0^M.341$	$-0^M.015$
92....	20 45.3	0.375	−0.005	100...	4 53.5	0.342	−0.014
93....	21 46.0	0.340	+0.020	101...	5 05.0	0.330	0.000
94....	23 10.6	0.367	−0.018	102...	5 19.7	0.341	−0.010
95....	23 27.4	0.331	+0.013	103...	5 49.0	0.357	−0.026
96....	23 50.2	0.351	−0.010	104...	6 12.2	0.320	+0.013
97....	2 0 46.0	0.357	−0.016	105...	6 36.0	0.318	+0.016
98....	1 19.3	0.309	+0.023	106...	6 53.6	0.369	−0.030

made by comparing the variable with B.D.$+61°493$, whose normal magnitude, 8.92, was determined by reference to three other stars in the same field, whose magnitudes are given in *Harvard Annals*, **64**. The observations were grouped into 106 normals. The period, as determined, is $2^d22^h14^m41^s.7$. The shape of the curve shows that the variation is of the β Lyrae type, as it is continuous, indicating that the stars are sensibly elliptical.

The ellipticity of the stars was determined by the graphical method and it was found that there was also a slight reflection effect. The observations were corrected for these and the rectified light-curve obtained. It was found that in the rectified light-curve the secondary eclipse had a constant phase for nearly five hours, while the primary eclipse was clearly round-bottomed. On account of the long, flat bottom, it is obvious that the eclipse at secondary minimum is total. The primary eclipse, from the shape of the curve, would seem to be partial, if the star-disks are of uniform brightness. For the secondary to be a total eclipse and the primary partial, the orbit would necessarily be highly eccentric. This, however, is not consistent with the dimensions of the stars, as they are large in relation to the size of the orbit. The two minima are of the same duration and the secondary comes at mid-period.

When a solution was attempted, assuming a circular orbit and uniform disks, with the secondary eclipse total and the primary annular, the computed curves for the two minima were not consistent with the shape of the observed curves. The deviations in many cases were several times the probable error of one observation.

The assumption of uniform brightness in the stellar disks was therefore abandoned.

In the case of stars darkened toward the edge, Professor Russell has computed several light-curves due to annular eclipses of darkened stars for different values of k (Fig. 1, *Astrophysical Journal*, **36**, 386, 1912) and has shown that the curves are all round-bottomed, since, as it moves in front of the larger star, the smaller one continues to cut off a brighter area of the larger star until the center is reached.

On carrying out the solution for the darkened disks, the secondary was considered total and the primary annular. From the rectified depths of the primary and secondary eclipses and the equation $Q(k,\alpha_0)=\frac{(1-\lambda_2)}{\alpha_0-(1-\lambda_1)}$, where $1-\lambda_1=0.299$ and $1-\lambda_2=0.171$, for total eclipse $\alpha_0=1$; $Q(k, \alpha_0)=0.360$. From Table V, for the value of function $Q(k, \alpha_0)=0.360$ there are possible values of k and α_0'' ranging from $k=0.552$ to $k=0.506$, and $\alpha_0''=1$ to $\alpha_0''=1+x$ (where the loss of light for central annular eclipse is $1+x$ times that at internal contact), which corresponds to grazing annular and central annular eclipse. It was found, after several trials (use being made of Table IIy) that the primary eclipse was best represented by a central annular eclipse with value of $\alpha_0''=1.147$ and $k=0.52$. The primary curve was then

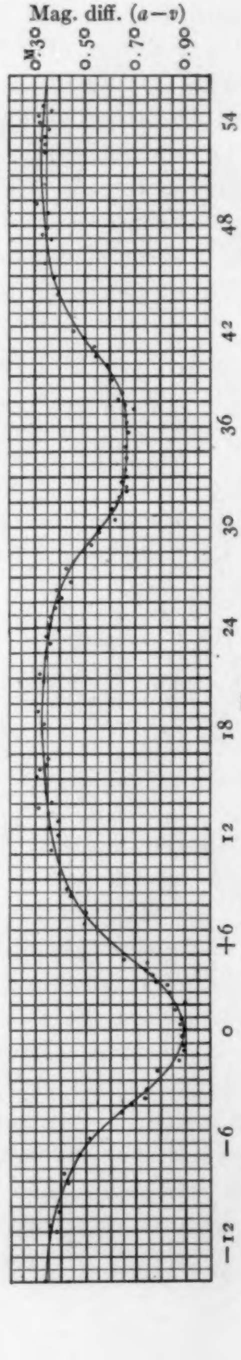

FIG. 5.—Mean light-curve of TX Cassiopeiae

computed by means of Table Iy, while Table IVx was used for the secondary eclipse. The computed constant phase for secondary eclipse is 4.7 hours in duration and satisfies the observations perfectly.

The elements were computed in the usual way and are given in the table at the end. The residuals have been scaled off from the computed curve and are given in the table of observations. The probable error of one normal place is $\pm 0^{\mathrm{M}}.011$. Fig. 5 gives the theoretical light-curve derived from the solution for stars darkened toward the edge, Fig. 6 diagrams showing stars at elongation and at time of primary eclipse.

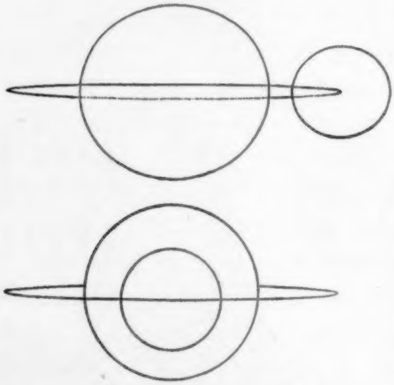

FIG. 6.—Elongation and primary eclipse for TX Cassiopeiae.

T LEONIS MINORIS

This variable was discovered on the Harvard plates by Miss Leavitt and announced as an Algol variable with a range of $2^{\mathrm{M}}.0$. A note was published by the writer in *Astronomische Nachrichten*,[1] concerning the variation of the star. The period was thought to be within a few minutes of $3^{\mathrm{d}}0^{\mathrm{h}}20^{\mathrm{m}}$ and the variation at least $1^{\mathrm{M}}.7$. From later observations, it was found that the actual period is $8^{\mathrm{m}}38^{\mathrm{s}}$ longer and the variation is nearly $2^{\mathrm{M}}.5$. The Harvard photographic measures were used, and in this case, as in others, have been of extreme value, combined with the visual observations, in establishing a definitive period. The series of observations covers twenty-five years, and during that interval there is no evidence of the period changing.

The light-curve for this system is not so well defined as in the other systems. Weather conditions have been unfavorable, and as the star is of the tenth magnitude, observations at the time of bright moonlight were rarely taken. In all, 265 observations (16 measures each) were made by comparing T Leonis Minoris

[1] **199**, 221, 1914.

TABLE VI

Table of Observations. T Leonis Minoris

PRIMARY MINIMUM

Normal No.	Phase	Mag. Diff.	O.−C.$_u$	O.−C.$_d$	Normal No.	Phase	Mag. Diff.	O.−C.$_u$	O.−C.$_d$
1..	$-5^{h}26^{m}.7$	$(v-a)$ $0^{M}.30$	$-0^{M}.06$	$-0^{M}.06$	16..	$-0^{h}51^{m}.7$	$(a-v)$ $1^{M}.43$	$0^{M}.00$	$0^{M}.00$
2..	4 54.0	0.27	−0.08	−0.09	17..	0 31.4	1.77	−0.02	0.00
3..	4 28.1	0.30	−0.03	−0.02	18..	−0 11.6	2.10	−0.03	−0.02
4..	3 37.1	0.25	+0.02	+0.02	19..	+0 07.5	1.70	+0.41*	+0.41
5..	3 28.1	0.19	+0.02	+0.01	20..	0 34.5	1.50	+0.09	−0.10
6..	3 14.9	0.18	+0.07	+0.05	21..	0 50.0	1.14	+0.16	−0.16
7..	2 50.8	0.09	+0.03	+0.02	22..	1 05.0	1.11	+0.05	+0.06
8..	2 39.4	$(a-v)$ 0.01	+0.06	+0.05	23..	1 29.6	0.88	−0.09	−0.09
9..	2 26.1	0.29	−0.07	−0.09	24..	1 44.6	0.54	−0.05	−0.04
10..	2 12.4	0.34	−0.03	−0.03	25..	2 00.8	0.39	+0.01	+0.01
11..	2 01.0	0.37	+0.01	0.00	26..	2 16.8	0.34	−0.06	−0.06
12..	1 48.9	0.53	0.00	−0.03	27..	2 37.5	0.18	−0.09	−0.09
13..	1 33.9	0.66	+0.01	0.00	28..	2 54.8	0.15	−0.16	−0.17
14..	1 19.4	0.91	0.00	+0.01	29..	3 10.0	$(v-a)$ 0.16	+0.06	+0.05
15..	1 01.7	1.30	−0.05	−0.09	30..	3 23.0	0.25	+0.10	+0.09

* Normal No. 19 half-weight.

CONSTANT LIGHT AND SECONDARY MINIMUM

Normal No.	Phase	Mag. Diff. $(v-a)$	O.−C.	Normal No.	Phase	Mag. Diff. $(v-a)$	O.−C.
1......	$0^{d}\ 6^{h}26^{m}.6$	$0^{M}.29$	$-0^{M}.07$	20......	$1^{d}13^{h}.28^{m}.1$	$0^{M}.36$	$+0^{M}.03$
2......	7 14.5	0.26	−0.10	21......	17 32.4	0.32	−0.04
3......	10 32.6	0.45	+0.10	22......	17 57.7	0.41	+0.05
4......	11 15.6	0.44	+0.08	23......	18 19.7	0.40	+0.04
5......	11 52.0	0.26	+0.01	24......	20 52.3	0.29	−0.07
6......	12 12.7	0.36	0.00	25......	21 24.9	0.33	−0.03
7......	17 56.6	0.43	+0.07	26......	21 52.0	0.33	−0.03
8......	23 20.7	0.45	+0.09	27......	22 21.0	0.30	−0.05
9......	1 0 38.7	0.39	+0.04	28......	22 40.3	0.34	−0.01
10......	7 45.2	0.30	−0.06	29......	24 45.6	0.35	−0.01
11......	8 46.9	0.36	0.00	30......	2 8 12.9	0.37	+0.01
12......	9 26.0	0.39	+0.03	31......	8 16.5	0.33	−0.03
13......	9 49.3	0.31	−0.04	32......	9 06.2	0.37	+0.01
14......	10 16.3	0.33	−0.01	33......	9 33.9	0.34	−0.02
15......	10 46.2	0.33	0.00	34......	10 23.0	0.43	+0.08
16......	11 36.5	0.28	−0.04	35......	14 48.5	0.30	−0.05
17......	11 59.9	0.36	+0.04	36......	16 34.2	0.39	+0.03
18......	12 19.7	0.34	+0.02	37......	17 25.5	0.40	+0.04
19......	12 39.1	0.29	−0.04	38......	18 16.5	0.30	−0.03

with B.D. 34°2032, magnitude 10.35 (magnitude of B.D. 34°2032 found with reference to T Leonis, whose magnitude was determined photographically at Harvard). These observations were grouped into 68 normals of from 2 to 5 observations each according to circumstances. Thirty of the normals are in constant light, and make T Leonis Minoris $0^{M}.357$ brighter than the comparison star. At

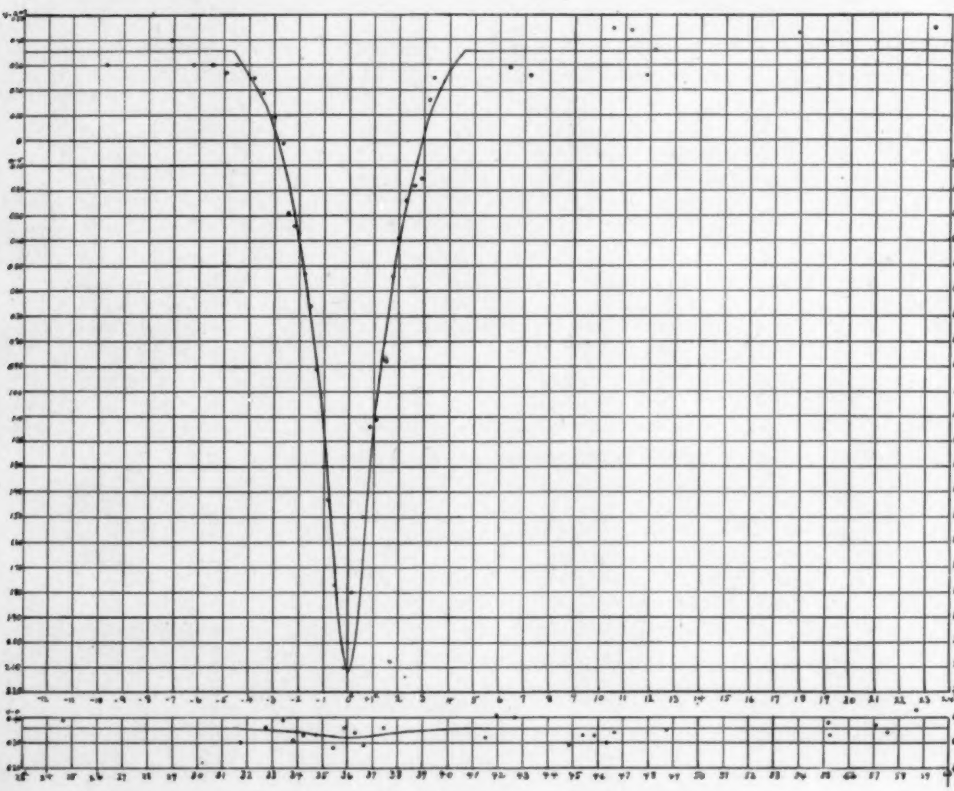

FIG. 7.—Mean light-curve of T Leonis Minoris

primary minimum T Leonis Minoris is $2^{M}.12$ fainter, losing $2^{M}.48$. The secondary eclipse has been observed three different times and appears to be $0^{M}.04 \pm 0^{M}.02$ in depth.

The light between eclipses remains sensibly constant. The primary eclipse is evidently partial. For the uniform solution, the curve which defines the primary minimum was well represented

by a value of the function $\chi(k, \alpha_0, \frac{1}{4}) = 1.999$. This value gave a series of values of k and α_0 ranging from $k=0.792$ to $k=0.848$ and from $\alpha_0=0.90$ to $\alpha_0=1.00$. To represent the observed depth of the secondary minimum k must be 0.812 and α_0, 0.95. The light-curve was computed by means of Table III*a* and the equation $\sin^2\theta(n) = C\omega_2(n) + D\omega_1(n)$ where $c=0.0759$ and $D=0.03796$.

For darkened disks, from the relation $Q(k, \alpha_0) = \dfrac{1-\lambda_2}{\alpha_0-(1-\lambda_1)}$ and the depths of primary and secondary minima, the values for k and α_0 range from 0.537 to 1.00 for k and from 0.93 to 1.00 for α_0. To represent the shape of the light-curve of primary minimum, it was found that k must be 0.935, whence $\alpha_0=0.935$. The curve was computed in the usual manner. The residuals for the two solutions are given in the table of observations. Fig. 7 gives the theoretical light-curve for uniform solution and Fig. 8 diagrams of the system for the two solutions.

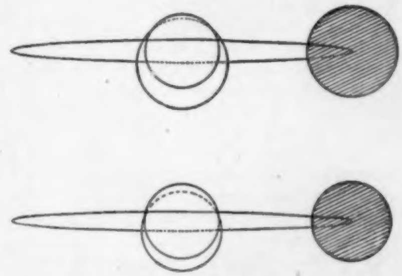

FIG. 8.—Diagrams resulting from solutions for uniform and darkened stars, T Leonis Minoris.

Table VIII gives the elements resulting from the solutions for the three stars TW, TX Cassiopeiae, and T Leonis Minoris.

The table of results contains, besides the elements as computed for the various solutions, densities corrected for polar flattening as well as for the probable difference of the masses of the brighter and fainter components, in accordance with the method outlined by Shapley in *Contribution No. 3, Princeton University Observatory*, p. 123. The hypothetical parallax, distance in light-years, size with respect to the sun, and the ratio of brightness of each system have been computed by the method of Russell and Shapley (*Astrophysical Journal*, **40**, 417, 1914), using their estimates of the mean mass and surface-brightness as a function of the spectral type. The principal components of three of the four systems are of quite normal density. Their estimated brightness is comparable with that of such stars as α Lyrae, and their distances are similar to those estimated for

most eclipsing variables. T Leonis Minoris, however, appears to be situated in space considerably farther from the Galactic plane than any of the 90 variables previously investigated. TX Cas-

TABLE VII

TABLE OF RESULTS

Elements of the System		TV Cassiopeiae	
		Uniform	Darkened
Maximum radius of brighter star.....	a_b	0.301	0.325
Minimum radius of brighter star.....	b_b	0.285	0.314
Maximum radius of fainter star......	a_f	0.285	0.287
Minimum radius of fainter star.......	b_f	0.271	0.277
Ratio of radii of the two stars........	k	0.95	0.884
Ratio of the axes of the spheroidal stars	$1+\frac{1}{2}z$	1.052	1.065
Least apparent distance of centers....	$\cos i$	0.1783	0.2720
Inclination of the orbit.............	i	79°45′	74°15′
Eccentricity of the orbit............	$e \sin \omega$		
	$e \cos \omega$	−0.010	−0.010
Maximum percentage loss of light at primary minimum...............	α_{op}	0.628	0.640
Maximum percentage loss of light at secondary minimum............	....	....	....
Difference of light of the sides of the fainter star....................	$2b$	0.074	0.074
Light of the brighter star...........	L_b	0.859	0.844
Light of the fainter star { Bright side ..	L_f	0.141	0.156
Light of the fainter star { Faint side ...	L_f-2b	0.067	0.082
Ratio of surface-brightness of the bright sides of the two stars...........	J_b/J_f	6.7	7.0
Ratio of surface-brightness of the sides of fainter star..................	$L_f/(L_f-2b)$	2.1	1.9
Density of brighter star............	ρ_b	0.118	0.089
Density of fainter star.............	ρ_f	0.056	0.054
Hypothetical radius of brighter star in solar radii......................	A	2.46	....
Absolute magnitude of brighter star...	M	0.3	....
Brightness compared with sun as unit..	Light	100	....
Hypothetical parallax..............	π	0".0031	....
Number of light-years..............	Distance in light-years	1.030	....
Probable diameter, sun=1..........	Probable diameter	3.0	....
Distance in light-years from Galactic plane..........................	....	1020	....
Distance projected on Galactic plane (in light-years).................	....	−53	....

siopeiae is remarkable for its low density, lower than that of any other star of spectrum B, except β Lyrae; its great estimated brightness, which, however, does not exceed that of some of the

stars of spectrum B in Kapteyn's group in Scorpius; and its enormous estimated distance of 10,800 light-years, which, however, would be greatly reduced by the assumption of even a small absorption of light in space.

TABLE VIII

TABLE OF RESULTS—*Continued*

Elements	TW Cassiopeiae		TX Cassiopeiae	T Leonis Minoris	
	Uniform	Darkened	Darkened	Uniform	Darkened
a_b	r_1 0.176	0.165	0.567	r_1 0.217	0.218
b_b	....	....	0.519	....	....
a_f	r_2 0.151	0.165	0.295	r_2 0.266	0.233
b_f	....	....	0.270		
k	0.858	1.00	0.520	0.812	0.935
$(1+\frac{1}{2}z)$	....	....	1.080	....	....
cos i	0.0058	0.0551	0.0442	0.0648	0.0566
i	89°40′	86°50′	82°30′	86°20′	86°45′
a_{op}	1.00	0.850	1.147	0.950	0.935
a_{os}	1.00	0.869	1.000	....	....
e sin ω	$e=0.040$	0.050	....	....	....
e cos ω	$\omega=274°$	273°	....	....	....
$2b$	....	....	0.030	....	....
L_b	0.592	0.582	0.829	0.945	0.961
L_f	0.408	0.418	0.171	0.055	0.039
(L_f-2b)	....	....	0.141	....	....
J_b/J_f	1.07	1.39	1.5	35.7	28.5
$L_f/(L_f-2b)$	....	....	1.2	....	....
ρ_b	0.167	0.185	0.0068	0.112	0.112
ρ_f	0.214	0.185	0.0214	0.018	0.026
A		1.88	5.61		1.98
M		+0.3	−3.1		+0.3
Light		60	1400		60
π		0".0019	0".0003		0".0013
Distance in light-years		1600	10,800		2600
Probable diameter		2.5	9.0		2.6
Distance in light-years from Galactic plane		170	750		2050
Distance projected on Galactic plane in light-years		1620	10,800		1600

This investigation has been carried out under the direction of Professor H. N. Russell, and I am indebted to him for valuable suggestions given in the course of the work.

PRINCETON UNIVERSITY OBSERVATORY
June 29, 1915

THE STRUCTURE OF THE THIRD CYANOGEN BAND AND THE ASSOCIATED TAILS

By H. S. Uhler and R. A. Patterson

The present investigation forms an essential part of an attempt to affirm or deny definitely Thiele's hypotheses concerning band spectra. The complete solution of this problem involves two distinct kinds of work, (*a*) experimental and (*b*) arithmetical. The results obtained by a careful study, both qualitative and quantitative, of the band at λ 3883 and of the tails of shorter wave-length, as radiated by the direct-current carbon arc in air at atmospheric pressure, are recorded in the later paragraphs of this paper. An article, by H. S. Uhler,[1] dealing with one phase of the computational side of the question has already been published. Historically, the experimental data should have been formally presented before the theoretical paper, but as the mathematical analysis was independent of any particular source of wave-lengths, as Leinen's numerical data for the carbon band at λ 5165 were used more extensively than the wave-lengths recorded below, and as the present account was not ready to go to press at the time of completion of the calculations, it was thought best not to defer the publication of the theoretical article until after the experimental results had appeared in print.

Formulae for the series lines of band spectra have been proposed by H. Deslandres, Kayser and Runge, G. Higgs, J. N. Thiele, A. Fabry, J. Halm, and W. Ritz. The equations of Halm, Higgs, and Ritz are special forms of the functions given by Deslandres, Thiele, and Kayser and Runge, respectively. Since the subject of the laws of all kinds of spectra has been presented very clearly and completely by H. Konen in a volume entitled *Das Leuchten der Gase und Dämpfe* (1913), as well as in Kayser's classic *Handbuch der Spectroscopie*, it would be superfluous to give bibliographic references and general historic details in this place. Suffice it to say that the most comprehensive hypotheses have been advanced by

[1] *Astrophysical Journal*, **42**, 72, 1915.

Thiele and subjected to experimental tests by their proposer and by several later investigators. Because of the general nature of these hypotheses and the fact that their validity has been accepted by some writers of note and rejected by others, it is a matter of no little importance to endeavor to obtain new evidence for them or against them. It should also be remarked, at this juncture, that the question of the origin of the bands between λ 2000 and λ 4217 is not germane to the present work, so that the usual terminology of "cyanogen" bands will be adhered to, notwithstanding the fact that W. Grotrian and C. Runge[1] have recently shown experimentally that the bands at λλ 3360, 3590, 3883, and 4216 are due to nitrogen alone and not to a nitro-carbon. Later experiments of the authors, who established a direct-current arc between copper electrodes in an atmosphere of nitrogen, which had been freed from carbon and carbon compounds, were consistent with the results obtained by Grotrian and Runge.

The law for series spectra in general was formulated by Thiele[2] as

$$\lambda=f[(n+c)^2] .$$

In the case of band spectra this function must embody the following characteristics, which are the only ones amenable to direct experimental investigation and independent of the calculation of the series "phase" c.

Every series must have a head ($n=0$) and a tail ($n=\pm\infty$). In any one series the intervals between successive lines must be finite near the head, must increase up to a maximum as the arithmetical value of n increases, and then decrease to zero as a limit as the extreme wave-length of the tail is approached. In other words, the lines should be discrete and theoretically resolvable in the region of the head, whereas an infinite number of lines should coincide to form the edge of the tail.

A concrete example of tail bands was first given by Thiele[3] himself, who called attention to the presence on the spectrograms of cyanogen, taken by Rydberg, of "certain sudden interruptions of the fogged gray background which might be regarded as the tails

[1] *Physikalische Zeitschrift*, **15**, 545, 1914.

[2] *Astrophysical Journal*, **6**, 66, 1897.

[3] *Ibid.*, **6**, 67, 1897.

of the series in this spectrum. . . . ." Subsequently, A. S. King[1] determined the wave-lengths of the edges of these bandlike structures, three of which are situated just below (on the frequency scale) the 3590 band. In addition to these, he recorded thirteen above λ 3590 and three on the less refrangible side of the 3883 head. He concluded that the bands under investigation belong to the cyanogen spectrum and constitute the tails of the series emanating from the several heads of the groups of bands commencing at λλ 3590, 3883, and 4216. The particular arrangement of heads and tails was established empirically. King did not attempt to determine the wave-lengths of the series lines which converge toward the tails.

In the year 1904, the same region of the cyanogen spectrum was studied anew by F. Jungbluth[2] in Kayser's laboratory. The main object of his investigation was to test the hypotheses of Thiele and the conclusions drawn by King. By using greater dispersion and resolving power than his predecessors,[3] and by making long exposures, Jungbluth tried to follow the chief series of the 3883 band from this head to one of the tail bands measured by King. The attempt did not meet with success because the lines of the series ceased, at λ 3640, to retain their characteristic, differentiating intensity, which is so noticeable for ordinal numbers less than about 168, and because the spectrum had too dense a structure above λ 3640 to enable the observer to extend the series by the aid of tentative extrapolation. For the same reasons, the second series could not be followed beyond the point mentioned, while the third and fourth series became indistinguishable at about λ 3680. In regard to the tails, Jungbluth pairs them with the heads in a different manner from that of King. He made no attempt either to verify the wave-lengths given by King, or to analyze the tails into series, or to apply Thiele's method of computation to the four apparent series.

In the year 1907, Professor H. Kayser called the attention of the senior author to the importance of repeating and extending Jung-

[1] *Astrophysical Journal*, **14**, 323, 1901. [2] *Ibid.*, **20**, 237, 1904.

[3] H. Kayser and C. Runge, *Abhandlungen der königlichen Akademie der Wissenschaften*, Berlin, 1889, Anhang.

bluth's work, in the hope that new light might be thrown on Thiele's hypotheses and on the laws of band structure in general. The work was commenced as soon as possible, but its progress was interrupted to such an extent that it could not be completed before the present year.

APPARATUS AND SPECTROGRAMS

The majority of the negatives of the 3590 and 3883 bands were taken at the Johns Hopkins University by H. S. Uhler. The third order of Rowland's original concave grating, which has a radius of curvature of 653 centimeters and 789 lines to the millimeter, was used exclusively. The best spectrograms of the more intense lines were obtained with fine-grained, lantern-slide plates manufactured to order by the M. A. Seed Dry Plate Co. With these plates the length of exposure varied from five minutes to two hours. The sensitive films had the peculiarity of developing more strongly near the edges than along the medial lines of the plates. This property was advantageous in picking out the spectral lines of different series. The region of the spectrum between λ 3590 and λ 3650 was so faint as to require four-hour exposures with "Seed 27" plates. In order to minimize the effects of vibrations of the building all of the long exposures were made between 12:30 and 5:00 A.M. The processes of development were carried out with great care and under the able guidance of L. E. Jewell. As source, a 110-volt direct-current arc was used. To facilitate the identification of impurity lines, negatives were taken with regraphitized Acheson graphite rods as well as with ordinary commercial carbons. For obvious reasons, the electrodes were always maintained horizontal, so that only the image of the violet center of the arc fell on the slit proper.

In addition to the spectrograms obtained in Baltimore, auxiliary ones were taken in this laboratory in the fourth order of one of Anderson's concave gratings. This grating has about the same radius of curvature as the one first mentioned, but three-fourths the number of lines per unit length. The individual rulings are unusually long, however, and the intensity of the spectra produced is correspondingly increased. All of the tails above λ 3590 were photographed in the second order of the new grating and the strongest ones were also recorded in the fourth order.

The wave-lengths have been calculated in the new international system, the secondary interferometer standards being employed directly whenever possible. In some instances the data given by Keivin Burns[1] were used and found to be very satisfactory. For sharp lines on clear background the wave-lengths are believed to be correct in absolute value to ±0.005 A, and in relative value to ± 0.002 A. Other lines, including the edges of the heads and tails, are probably accurate to within ±0.01 A.

EXPERIMENTAL RESULTS

All the lines, between λ 3590 and λ 3883, except those too faint to be set upon even approximately, which are radiated by the carbon arc in air, have been measured and their intensities *relative to neighboring lines* estimated. Also many of the lines have been definitely assigned to certain series. These results are given in Table I. The first and second columns contain respectively the wave-lengths in dry air at 76 cm pressure, and the symbols indicating the series and character of the lines. The key to the notation follows:

A_1 = belongs to singlet series from first head
A_2 = belongs to doublet series from first head = returning P branch of open doublets
B_1 = belongs to singlet series from second head
B_2 = belongs to doublet series from second head
C_1 = belongs to singlet series from third head
D_1 = corresponds to Jungbluth's IV series
D_2 = corresponds to second branch of Jungbluth's IV series
E = corresponds probably to Jungbluth's "fifth series"

The subscripts on the foregoing letters designate different branches which are not definitely continuous.

I = very intense	F = very faint
i = intense	b = broad, diffuse
m = medium	d = probably double
w = weak	c = confused with
f = faint	s = superposed upon

H_1, H_2, H_3, H_4 = first four heads of 3883 band
H'_1, H'_2, H'_3 = first three heads of 3590 band
T_1, T_2, T_3 . . . . = "tails"

[1] *Lick Observatory Bulletin*, No. 247, p. 27, 1913.

TABLE I

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3883.402	H_1	3876.843	A_1w	3868.487	B_2f	3861.711	Ib
.191	A_2i	.481	A_2i 55	.407	A_1msB_2	.567	A_1IbdsH_3
.102	A_2i	.415	A_2i	.124	B_1w	.543	f
2.990	A_1i	.315	A_1f P	.033	B_2f	.456	f
.853	A_1i	5.939	A_2i 56	7.963	B_2f	.317	f
.752	A_2f	.873	A_2i	.860	A_2f	.264	B_1f
.697	A_1i	.772	A_1F P	.779	A_2msB_1	.191	B_2w
.580	A_2w	.375	A_2i 57	.681	B_2F	.132	B_2w
.521	A_1i	.310	A_2i	.619	A_1isB_2	.026	w
.387	A_2w	4.791	A_2i 58	.384	B_1f	0.992	w
.321	A_1i	.727	A_2i	.302	B_2f	.916	f
.171	A_2m	.602	A_1F K=0	.234	B_2f	.829	mb
.101	A_1i	.190	A_2i 59	.062	A_2f	.626	A_1IsB_1
1.938	A_2m	.123	A_2i	6.982	A_2msB_1	.497	B_2m
.875	A_1I	.004	A_1f 1	.895	B_2f	.424	B_2m
.682	A_2i	3.567	A_2i 60	.816	A_1isB_2	.276	w
.616	A_2i	.501	A_2i	.555	B_1f	.221	ms
.587	A_1i	.371	A_1w 2	.471	B_2w	.042	f
.403	A_2i	2.965	A_2m	.396	B_2w	59.957	$B_1$is
.346	A_2i	.739	A_1i 3	.240	A_2w	.845	B_2w blurr
.305	A_1i	.252	A_2i 61	.168	A_2w	.783	B_2f
.106	A_2i	.180	A_2i	.108	B_1f	.670	A_1Id
.051	A_2i	.057	A_1m 4	5.991	A_1isB_2	.517	w
0.999	A_1i	1.566	A_2i 62	.648	B_1f	.424	wb
.791	A_2i	.501	A_2i	.563	B_2w	.277	B_1wc
.731	A_2i	....	sH_2 head of 1-1 band	.492	B_2w	.204	B_2w
.670	A_1i	.441	H_2	.399	A_2w	.115	B_2w
.450	A_2i	.239	B_1m	.327	A_2w	8.993	w
.392	A_2i	.133	B_1m	.151	A_1isB_1	.918	m
.327	A_1i	.010	B_1m	.085	B_2w	.836	f
.092	A_2i 47	0.876	A_2isB_1	.010	B_2w	.789	f
.033	A_2i	.808	A_2w	4.888	F	.683	A_1id
79.964	A_1i P	.719	B_1m	.667	B_1f	.591	B_1m
.712	A_2i 48	.665	A_1msB_2 6	.595	A_2mcB_2	.515	B_2w
.654	A_2i	.550	B_1m		blurr	.450	B_2w
.578	A_1i	.481	B_2f	.458	A_2mcB_2	.256	w
.311	A_2i 49	.358	B_1m	.300	A_1i	.180	w
.253	A_2i	.283	B_2f	.123	B_1f	.098	w
.183	A_1i	.146	A_2isB_1	.062	B_2w	7.99	w
8.891	A_2i 50	.069	A_2isB_2	3.993	B_2w	.896	B_1mc
.828	A_2i	69.921	A_1ibsB_1 7	.661	A_2w	.814	B_2w
.749	A_1i P	.831	B_2f	.593	A_2wsB_1	.687	A_1id 21 = K''
.577	F*	.667	B_1w	.528	B_2w	.530	w
.448	A_2i 51	.580	B_2f	.390	A_1ibcB_2	.449	w
.389	A_2i	.410	A_2isB_1	2.976	B_2w	.334	wb
.303	A_1m	.331	A_2isB_2	.900	B_2w	.158	B_1mc
7.989	A_2i 52	.180	A_1m 8	.768	A_2w	.074	B_2f blurr
.925	A_2i	....	B_1w blurr	.694	A_2w	6.994	B_2f
.832	A_1m	.066	B_2w	.489	A_1ibsB_1	.922	m
.506	A_2i 53	.021	B_2w	.403	B_2w	.658	A_1id 22
.446	A_2i	8.829	B_1w	.324	B_2w	.516	f
.351	A_1w P	.721	B_2w	.107	F	.407	B_1mc
.005	A_2i 54	.645	A_2f	1.955	f	.314	B_2f
6.938	A_2i	.571	A_2msB_1	.854	H_3	.234	B_2w

* Foreign ?

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
....	F	3849.498	f	3843.831	f	3836.880	D$_1$f
3856.056	f	.423	w	.754	f	....	F
5.964	w	.335	m	.691	f	....	F
.883	m	.260	f	.457	mb	.540	A$_1$IsB$_1$C$_1$
.791	f	.008	A$_1$id	....	f	.494	A$_1$IsB$_1$C$_1$
.622	A$_1$IsB$_1$	8.844	B$_1$mb	....	f	.118	D$_1$w
.423	B$_2$w	.697	f	.009	A$_1$IsB$_1$	....	F
.347	B$_2$w	.612	w	2.973	A$_1$IsB$_1$	5.839	f
.123	w	.536	f	.647	w	.772	w
4.932	w	....	F	.464	f	....	F
.851	B$_1$ms	....	f	.402	F	.549	C$_1$w
.744	H$_4$	....	f	.252	f	.389	B$_1$m
.662	w	.194	w	.182	f	.343	B$_1$msD$_1$
.566	A$_1$IbdsH$_4$	.104	f	1.953	B$_1$md	.202	A$_1$i
.305	w$_{\text{blurr}}$	.052	f	.753	A$_1$i	.147	A$_1$i
.252	f	.92	B$_1$mbcA$_1$	.710	A$_1$i	....	F
.199	f	7.839	A$_1$idcB$_1$	....	f	4.897	f
.144	f	....	F	.477	w	.824	f
.062	B$_1$m	....	F	.392	w	....	F
3.909	w$_{\text{blurr}}$	....	F	.154	f	.635	f
.812	w	....	F	0.994	C$_1$wcB$_1$	.573	C$_1$wsD$_1$
.748	f	....	F	.902	B$_1$mdc	....	F
.686	f	.269	m	....	F	....	F
.570	fc	....	f	....	F	.232	B$_1$m
.487	A$_1$id	....	fcB$_1$	.479	A$_1$i	.184	B$_1$m
.380	mb	6.965	B$_1$mc	.436	A$_1$i	.036	f
.220	B$_1$mb	.823	fb	.105	C$_1$w	3.950	f
2.908	w	.677	A$_1$i	....	F	.842	A$_1$i
.774	f	.633	A$_1$i	....	F	.785	A$_1$isD$_1$
.706	m	.537	f	39.829	B$_1$md	.611	C$_1$m
.554	f	.328	f	.739	f	.160	f
.391	A$_1$IdsB$_1$	.266	f	.669	f	.062	B$_1$m
.190	w	....	F	....	F	.016	B$_1$m
.121	w	....	F	.497	F	2.897	D$_1$fb
1.857	w	.000	B$_1$m	.437	F	.639	C$_1$m
.681	m	5.919	w	.346	f	.458	A$_1$i
.592	m	.832	f	.189	A$_1$isC$_1$	.404	A$_1$i
.527	B$_1$m	.797	f	.141	A$_1$ic	.173	Fb
.285	A$_1$id	.468	A$_1$i	8.759	B$_1$m	.054	D$_1$fbd
.168	w	.427	A$_1$i	.719	B$_1$m	1.875	B$_1$mc
.082	f	.320	m	....	F	.818	B$_1$mc
0.937	f	.214	f	.457	f	.753	wcB$_1$
.851	m	.018	B$_1$is	.349	C$_1$mc	.644	C$_1$w
.739	w	....	f	....	F	.356	f
.647	B$_1$m	4.846	f	....	F	.187	D$_1$fb
.525	mb	.715	w	7.877	A$_1$i	.064	A$_1$i
.405	w	.643	f	.825	A$_1$i	.005	A$_1$i
.302	w	.574	f	.651	B$_1$m	0.822	C$_1$w
.158	A$_1$id	.497	f	.614	B$_1$m	.665	B$_1$m
.056	F	.429	f	.420	C$_1$w	.611	B$_1$m
49.964	w	.352	f	....	F	.434	F
.876	w	.250	A$_1$i	....	F	.366	C$_1$wsD$_1$
.749	B$_1$mb	.206	A$_1$i	.152	f	.295	F
.646	F	.013	B$_1$m	.083	f	.073	f

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3829.901	F	3823.354	F	3816.547	D_1f	....	F
.796	F	.267	F	.278	B_1m	3810.004	w
.650	A_1i	.083	B_1m	.201	B_1mcA_1	09.913	D_1f
.588	A_1i 44	.024	B_1m	.170	A_1icB_1 53	.844	D_1f
.494	C_1F	2.953	C_1w	.101	A_1i	.755	A_1i 57
.441	B_1msD_1	.879	C_1wsD_1	5.831	C_1w	.690	A_1i
.390	B_1msD_1	.839	D_1f	.757	C_1w	.493	C_1w
.095	F	....	f	.525	D_1f	.420	C_1w
8.967	F	.645	w	.462	D_1f	.154	B_1m
.879	F	.557	F	.354	F	.084	B_1m
.811	F	.481	F	.236	f	8.997	f
.673	f	.322	A_1i	.079	f	.747	D_1f
.562	f	.259	A_1i 49	4.894	B_1m	.674	D_1f
.502	C_1f	.089	F blurr	.827	B_1m	.637	f
.454	D_1f	.019	F	.587	A_1IsC_1 54	.421	F
.399	D_1f	1.812	B_1mc$C_1$$D_1$	.517	A_1IsC_1	.337	F
.211	A_1IsB_1 45	.728	B_1Ics$C_1$$D_1$	.333	D_1f	.150	A_1IbsC_1 58
.155	A_1IsB_1	.422	F	.244	f	7.941	A_1I
7.976	f	....	F	.026	f	.774	f
.723	F	.190	f	3.939	f	.684	B_1m
.639	D_1f	0.991	f	.835	f	.602	B_1msD_1
.577	D_1f	.807	A_1isD_1	.637	f	.496	D_1w
.372	C_1w	.746	A_1isD_1 50	.558	f	.390	f
.206	Fbd	.637	C_1w	.485	B_1m	.298	F
.103	F	.567	C_1w	.431	B_1m	.154	f
6.942	B_1m	.421	B_1m	.331	C_1msD_1	.068	F
.886	B_1m	.378	B_1m	.258	C_1msD_1	6.861	C_1w
.770	A_1icD_1 46	.159	F	.119	F	.786	C_1w
.708	A_1isD_1	.086	F	2.984	A_1i 55	.643	f
.581	F	19.925	w	.923	A_1i	.440	A_1isD_1 59
.490	F	.783	D_1f	.807	f	.377	A_1isD_1
.307	C_1w	.724	D_1f	.655	F	.191	B_1m
.190	C_1w	.457	C_1w	.572	f	.120	B_1m
.073	Fb	.384	C_1w	.441	F	.035	F
5.986	F	.278	A_1i 51	.342	F	5.963	F
.915	F	.213	A_1i	.282	F	.850	f
.761	D_1F	.063	B_1id	.193	D_1f	.729	f
.674	B_1msD_1	8.739	D_1f	.121	D_1f	.524	C_1m
.616	B_1m	.670	D_1w	.062	B_1isC_1	.449	C_1m
.307	A_1i 47	.521	F	1.985	B_1isC_1	.191	D_1w
.239	A_1icC_1	.450	F	.720	F	.128	D_1w
.224	C_1wcA_1	.373	F	.650	F	4.932	fd
.125	C_1w	.262	C_1w	.382	A_1i 56	.774	A_1i 60
.033	F	.193	C_1w	.315	$A_1$is	.696	A_1IsB_1
4.946	F	.080	fb	.180	F	.619	B_1m
.795	D_1f	7.840	B_1m	.056	D_1f	.552	F
.739	D_1f	.732	A_1isD_1 52	0.992	D_1f	.462	f
.385	B_1m	.670	A_1isD_1	.912	F	.358	F
.325	B_1m	.384	B_1m	.789	C_1w	.285	F
.085	C_1w	.142	f	.712	C_1w	.176	C_1w
.017	C_1w	.055	C_1w	.615	B_1m	.098	C_1w
3.823	A_1isD_1 48	6.985	C_1w	.542	B_1m	3.992	D_1f
.760	A_1isD_1	.720	F	.332	F	.928	D_1f
.533	f	.622	D_1f	.248	F	.680	f

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3803.570	w	3797.234	C_1w	3790.535	B_1m	3783.905	B_1IsC_1E
.460	fb	.154	C_1w	.465	B_1m	.819	B_1IsC_1E
.174	B_1m	.057	Ef	.310	F	.541	A_1i
.090	A_1IsB_1 61	6.962	B_1msE	.230	F	.465	A_1i 72
.013	A_1i	.890	B_1m	.132	F	.093	D'_1w
2.808	C_1msD_1	.730	F	89.985	C_1w	2.998	D'_1w
.727	C_1msD_1	.658	F	.916	C_1w	.870	F
.577	w	.589	fc	.879	Ew	.799	F
.486	w	.555	fc	.780	Ew	.670	Ef
.333	f	.306	w	.590	F	.582	Ew
.214	f	.184	A_1i	.515	F	.395	C_1w
.060	F	.104	A_1i 65	....	F	.310	C_1w
1.900	f	....	F	.273	F	.213	B_1m
.820	fb	....	F	.193	F	.134	B_1m
.740	f	5.805	C_1msE	.040	A_1i	1.685	A_1isD'_1
.643	B_1m	.722	C_1wbcE	8.968	A_1i 69	.605	A_1isD'_1 73
.569	B_1mcD_1	.377	B_1m	.892	B_1m	.438	Ef
.503	D_1wcC_1	.300	B_1m	.816	B_1m	.292	Fb
.444	C_1wcD_1	.085	w	.696	Ef	.175	Fb
.375	A_1isC_1	4.991	w	.610	Ef	.063	F
.308	A_1i 62	.844	Fd	.526	C_1ms	.005	F
.146	F	.621	Ef	.456	C_1mcs	0.862	C_1w
.045	F	.531	Ef	.207	F	.756	C_1w
0.932	F	.417	A_1icC_1	.058	F	.509	B_1mc
.873	F	.351	A_1icC_1 66	7.959	f	.434	B_1mc
.683	F	....	mcA_1	.823	f	.314	D'_1fsE
.580	f	.117	F	.701	F	.227	D'_1fsE
.500	f	.049	F	.489	Ef	79.806	A_1i
.311	Em	3.914	F	.406	Ef	.732	A_1i 74
.248	F	.787	$B_1$is	.228	A_1IcsB_1	.599	F
.179	F	.699	$B_1$is	.155	A_1IcsB_1 70	.492	f
.098	B_1m	.447	Ef	6.906	C_1f	.394	f
.032	B_1isC_1	.358	Ef	.777	C_1f	.314	C_1w
799.974	C_1m	.158	F	.510	Fd	.219	C_1wc
.663	A_1i	.088	F	.283	Ef	.085	F
.587	A_1i 63	2.922	C_1w	.202	Ef	8.916	D'_1w
.217	Ew	.840	C_1w	5.976	F	.808	B_1isD'_1
.134	Ew	.639	A_1i	.898	F	.730	B_1m
8.915	f	.565	A_1icd 67	.832	D'_1w	.467	f
.860	f	.389	w	.733	D'_1w	.420	f
.761	f	.272	Ef	.574	B_1m	.345	f
.647	C_1w	.164	B_1isE	.495	B_1m	.155	f
.552	B_1ibsC_1	.083	$B_1$is	.390	A_1isC_1	.089	f
.459	B_1m	1.904	fb	.319	A_1IsC_1 71	7.919	A_1i
.326	F	.805	F	.248	w	.842	A_1i 75
.225	F	.724	F	.074	Ew	.755	C_1w
.158	F	.579	fb	4.993	Ew	.664	C_1w
.083	Ef	.473	C_1w	.799	f	.509	D'_1w
7.986	Efc	.386	C_1w	.678	f	.403	D'_1msE'
.926	A_1ic	.196	w	.464	D'_1m	.272	E'f
.852	A_1i 64	.085	Ews	.371	D'_1m	.092	B_1ib
.676	f	0.996	Ef	.221	F	....	F
.543	fb	.846	A_1i	.129	F	....	F
.372	f	.771	A_1i 68	.063	F	6.335	F

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3776.187	C_1wsE'	3768.302	A_1i	3760.363	A_1i 84	3750.808	E''F
.098	C_1msD$'_1$E'	.247	A_1IsB$_1$ 80	.290	A_1i	.702	E''F
.015	A_1isD$'_1$ 76	.166	B_1isC$_1$	59.976	C_1wdcD$''_1$	.206	A_1i
5.944	A_1i	.100	C_1w	.898	C_1wcD$''_1$	.134	A_1i
.515	F	7.949	F	.727	fbd	49.891	B_1mcsC$_1$
.446	F	.791	F	.180	B_1m	.788	B_1icC$_1$
.287	B_1m	.649	f	.103	B_1m	.587	D''_1fsE''
.204	B_1m	.540	D'_1m	8.546	D''_1w	.502	D''_1fsE''
4.979	E'f	.402	E'f	.452	D''_1f	8.838	F
.812	E'f	.301	E'f	.348	A_1iscC$_1$ 85	.313	E''F
.677	D'_1f	....	F	.287	A_1iscC$_1$	.215	E''F
.596	C_1msD$'_1$	....	F	....	F	.141	A_1isC$_1$D$''_1$
.515	C_1w	....	F	....	F	.068	A_1isC$_1$D$''_1$
.279	f	6.799	F	7.503	f	7.993	B_1m
.107	A_1i 77	.666	F	.446	f	.916	B_1m
.030	A_1i	.558	C_1w	.337	B_1m	.072	E''f
3.728	E'f	.462	B_1isC$_1$	.260	B_1m	6.987	E''f
.556	B_1msE'	.378	B_1m	.064	D''_1f	.608	D''_1f
.475	B_1m	.322	A_1i 81	6.968	D''_1f	.513	D''_1f
.244	D'_1f	.247	A_1i	.657	C_1w	.437	C_1w
.156	D'_1f	.127	E'f	.589	C_1w	.355	C_1w
.010	C_1w	.031	E'f	.332	A_1i 86	.075	A_1IcsB$_1$
2.925	C_1w	5.790	Fb	.263	A_1i	.004	A_1IcsB$_1$
....	F	.667	Fb	.003	f	5.845	E''f
....	F	....	F	5.878	f	.755	E''f
.472	E'f	.299	w	.569	D''_1ws	.098	D''_1w
.358	E'f	4.923	C_1w	.484	B_1isD$''_1$	.028	D''_1w
.180	A_1i 78	.843	C_1wsE'	.406	B_1m	4.728	C_1m
.106	A_1i	.764	E'F	.285	C_1w	.640	C_1msE''
1.805	B_1isD$'_1$	.653	B_1m	.149	C_1m	.544	E''F
.724	B_1isD$'_1$	.571	B_1m	4.511	E''f	.102	B_1m
.413	C_1w	.350	A_1i 82	.394	E''f	.041	B_1m
.325	C_1w	.277	A_1i	.300	A_1i 87	3.980	A_1i
.208	E'f	3.789	F	.227	A_1i	.910	A_1i
.097	E'fb	.681	F	.085	D''_1f	.624	D''_1fbd
0.984	F	.584	E'f	3.994	D''_1f	.426	D_2fsE''
.859	F	.487	E'f	.623	B_1m	.343	D_2fsE''
.729	F	.283	C_1w	.544	B_1m	.009	C_1w
.593	fb	.196	C_1w	.372	F	2.929	C_1w
....	F	2.965	D''_1f	.289	F	.255	B_1msE''
.240	A_1icsD$'_1$ 79	.845	B_1icsD$''_1$	.199	C_1wsE''	.181	B_1msE''
.169	A_1icD$'_1$	.760	B_1m	.109	C_1wsE''	1.887	A_1isD$_2$
.029	B_1m	.362	A_1isE' 83	2.586	D''_1f	.817	A_1isD$_2$
69.947	B_1msE'	.290	A_1isE'	.498	D''_1f	.707	f
.804	C_1wsE'	....	F	.258	A_1i	.658	f
.714	C_1w	1.829	F	.187	A_1i	.287	C_1w
....	F	.734	F	.044	E''F	.209	C_1w
....	F	.631	C_1w	1.937	E''F	.077	E''f
....	F	.547	C_1wc	.750	B_1m	0.485	D_2f
8.933	D'_1f	.514	D''_1fc	.675	B_1m	.441	D_2fc
.883	D'_1f	.352	D''_1fb	.536	C_1w	.349	B_1m
.793	F	.016	B_1m	.452	C_1w	.274	B_1m
.670	E'fb	0.938	B_1m	.089	D''_1f	39.783	A_1i
.570	E'f	.792	F	.007	D''_1ws	.716	A_1i

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3739.564	C_1w	3730.380	F	3718.690	C_1w	3707.252	A_1i
.483	C_1w	.309	F	.616	C_1w	.191	A_1i
.154	f	.091	D_2w	.458	D_2'f	6.944	B_1m
.014	D_2f	.016	D_2w	.382	D_2'f	.882	B_1m
8.926	D_2f	29.875	F	.273	A_1i	.588	C_1w
.520	F	.809	F	.210	A_1i	.522	C_1w
.426	B_1ms	.130	A_1i	7.65	F	.255	D_2'f
.347	B_1m	.064	A_1IsC_1	.54	F	.195	D_2'f
7.832	C_1w	8.994	C_1w	.32	F	.034	f
.756	C_1w	.697	B_1m	.28	F	5.031	A_1i
.672	A_1i	.633	B_1isD_2	.048	D_2'f	4.967	A_1IsB_1D_2'
.604	A_1i	.540	D_2f	6.960	C_1wsD_2'	.888	B_1msC_1D_2'
.527	D_2f	7.469	f	.877	B_1isC_1	.817	C_1w
.424	D_2fbc	.384	C_1ws	.806	B_1m	3.735	D_2'f
.296	f	.302	C_1w	.083	A_1i	.677	D_2'f
6.915	fb	.150	D_2f	.020	A_1i	.180	C_1w
.494	B_1m	.076	D_2f	5.654	D_2'f	.120	C_1w
.420	B_1m	6.975	A_1i	.586	D_2'f	2.949	B_1m
....	F	.906	A_1i	.217	C_1w	.899	B_1m
.103	C_1w	.747	B_1m	.149	C_1w	.805	A_1i
.029	C_1msD_2	.678	B_1m	4.893	B_1m	.750	A_1i
5.947	D_2f	.552	F	.828	B_1m	.470	D_2'fb
.612	A_1i	.479	F	.516	Fb	1.492	C_1w
.444	A_1i	5.703	D_2fc	.273	D_2'f	.429	C_1w
.207	f	.646	C_1mcD_2	.203	D_2'f	.305	D_2'f
.124	f	.576	C_1wc	.073	Fb	.263	D_2'f
4.553	B_1msD_2	4.908	fc	3.985	F	0.968	B_1m
.471	B_1msD_2	.804	A_1IsB_1	.884	A_1i	.915	B_1m
.290	C_1m	.732	A_1IsB_1	.823	A_1i	.575	A_1i
.006	F	....	F	.487	C_1w	.520	A_1i
3.933	F	....	F	.423	C_1w	.313	f
.625	f	3.907	C_1w	2.907	B_1msD_2'	699.805	C_1w
.417	A_1i	.831	C_1w	.844	B_1msD_2'	.752	C_1w
.344	A_1i	.656	f	.661	f	8.986	B_1msD_2''
....	F	.152	f	1.764	C_1w	.933	B_1msD_2''
.051	D_2f	2.815	B_1m	.682	A_1IsC_1	.342	A_1i
2.971	D_2f	.741	B_1m	.618	A_1i	.287	A_1i
.902	F	.639	A_1i	.505	D_2'f	.136	C_1w
.824	F	.572	A_1i	0.920	B_1m	.077	C_1w
.617	B_1isC_1	.165	C_1w	.857	B_1m	7.817	D_2''f
.544	B_1isC_1	.092	C_1w	.565	f	.778	D_2''f
.172	F	1.922	fb	.275	D_2'wb	6.974	B_1m
.106	F	.296	D_2'f	09.837	fbd	.923	B_1m
1.809	f	.219	D_2'fc	.470	A_1i	.685	D_2''fd
.731	f	.193	Fc	.411	A_1i	.467	C_1w
.569	D_2f	.073	F	8.935	B_1m	.416	C_1w
.484	D_2f	0.843	B_1m	.867	B_1isD_2'	.104	A_1i
.276	A_1i	.777	B_1m	.769	D_2'f	.053	A_1i
.208	A_1i	.457	A_1icsC_1	.293	C_1w	5.620	D_2''fd
0.922	C_1w	.399	A_1icsC_1	.228	C_1w	.002	B_1m
.863	C_1w	19.88	D_2'fc	7.910	F	4.951	B_1m
.747	f	.799	D_2'f	.825	F	.809	C_1wd
.663	B_1m	.451	Fb	.540	D_2'f	.592	D_2''fd
.590	B_1m	8.894	B_1ib	.483	D_2'f	3.863	A_1i

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3693.813	A_1i	3679.327	f	3663.791	Fbd	3652.217	F
.581	D''_2w	.232	B_1m	....	F	.090	fbd
.144	C_1w	.191	B_1m	.093	Fbd	1.956	F
.095	C_1w	8.994	C_1wd	2.909	F	.474	f
.018	B_1m	....	F	.866	F	.256	A_1mbcdB_1
2.968	B_1m	....	F	.375	A_1md	.113	F
.575	D''_2f	....	F	.166	F	.030	F
1.622	A_1i	.116	A_1i	.073	B_1w	....	Fbd
.570	A_1icD''_2	.080	A_1i	1.997	f	0.690	F
.472	C_1fcd	....	F	.911	F	.595	F
.038	B_1m	....	F	....	F	.275	F
0.987	B_1m	7.628	f	....	F	.182	w
.763	D''_2f	.513	C_1fbd	....	F	.036	F
89.901	C_1w	.291	B_1m	0.976	f	49.939	F
.858	C_1w	.255	B_1m	....	F	.720	F
.372	A_1i	....	F	.365	F	.587	B_1F
.327	A_1i	....	F	.246	B_1w	.506	B_1F
.061	B_1m	....	F	.140	A_1m	.387	F
.009	B_1m	6.074	C_1fd	....	Fb	.290	fc
8.283	C_1fd	5.864	A_1m	....	Fb	.255	fc
7.122	A_1i	.830	A_1m	59.506	fbd	.055	A_1w
.082	A_1IsB_1	.336	B_1wd	.317	f	....	F
.039	B_1i	4.636	C_1f	....	F	8.630	fb
6.931	f	3.614	A_1m	8.960	fbd	.404	f
.702	C_1w	.577	A_1m	.732	F	.346	f
.669	C_1w	.412	B_1wd	.654	F	....	F
....	F	.242	C_1f	.413	B_1w	7.843	B_1F
....	F	2.937	f	.086	T_1fb	.523	f
....	F	.602	F	7.910	A_1mbd	.468	f
5.115	B_1icsC_1	.368	Fb	....	F	6.856	A_1w
.077	B_1icsC_1	1.829	C_1fb	.147	f	.671	fb
4.872	A_1i	.682	f	....	F	.601	fb
.832	A_1i	.493	B_1wd	6.916	fc	.192	B_1wb
.339	fb	.368	A_1m	.600	B_1w	5.826	fb
....	F	.335	A_1m	.390	f	.736	fb
....	F	0.499	C_1f	.308	f	.303	F
3.792	fb	69.605	B_1mdc	5.678	A_1mbd	.113	F
.577	C_1w	....	F	.549	f	4.997	f
.544	C_1w	.099	A_1mdsC_1	.383	f	.904	F
.438	f	....	F	.187	f	.668	A_1fb
.148	B_1m	....	F	4.852	fc	.561	B_1F
.113	B_1m	7.897	f	.835	B_1fc	.181	fb
2.622	A_1i	.690	B_1wd	.706	F	.070	fb
.582	A_1i	....	F	.310	F	3.376	F
.172	C_1w	.037	f	.259	F	.276	F
1.764	C_1wb	6.852	A_1md	.179	f	.192	F
.192	B_1m	.648	f	.087	F	2.938	B_1f
.153	B_1m	.533	f	3.455	A_1mc	.582	F
.083	f	5.804	B_1w	.362	f	.477	A_1wb
....	F	.599	f	....	F	.111	F
0.511	C_1w	.423	f	....	F	1.806	f
.470	C_1w	4.615	A_1mbd	.075	B_1fcd	.691	f
.370	A_1i	.244	F	2.732	F	.334	B_1F
.331	A_1i	3.942	B_1w	.659	F	.039	F

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3640.973	F	3628.457	w	3617.435	w	3610.008	w
.908	F	.320	w	.058	w	09.729	w
.296	A_1fb	7.998	w	6.920	w	.688	w
.141	f	.746	w	.871	w	.569	w
39.532	f	.594	B_1w	.446	w	.525	f
.383	f	.418	wb	.350	f	.349	w
.215	f	.039	w	.318	mc	.239	w
.076	f	6.846	mb	5.828	w	.174	w
8.794	f	.563	F	.760	f	.072	f
.626	f	.332	w	.701	f	.019	w
.164	B_1F	.273	w	.242	ibd	8.754	w
.070	fb	.121	B_1w	.070	w	.681	m
7.865	f	5.710	w	4.729	w	.567	F
.328	f	.632	w	.674	w	.529	f
.189	F	.518	F	.642	w	.344	w
.097	fb	.435	F	.332	w	.263	w
6.743	F	.237	w	.196	w	.195	w
.609	B_1F	.160	w	.145	w	.038	m
.304	F	4.869	w	.000	w	7.789	m
.199	wb	.757	f	3.916	f	.717	m
.025	F	.612	w	.689	wb	.578	F
5.872	F	.436	F	.607	wb	.547	F
.314	fb	.263	F	.298	w	.437	F
.086	B_1f	.046	ib	.256	w	.313	m blurr
.045	B_1f	3.886	w	.153	w	.230	mc
4.965	F	.531	w	.087	m	6.837	mc
.537	F	.289	w	2.749	F	.765	f
.488	F	2.984	w	.621	w	.675	m
.408	F	.639	f	.575	m	.595	fcd
.094	F	.560	f	.494	f	.499	F
.013	F	.484	F	.426	w	.381	m
3.885	f	.406	w	.359	F	.301	w
.734	f	.029	f	.104	w	.257	f
.667	F	1.978	f	.051	w	.114	fbd
.308	f	.688	w	1.959	w	5.919	m
2.941	wc	.576	f	.878	w	.838	w
.867	wc	.461	w	.721	f	.786	f
.672	f	.020	w	.680	f	.636	fb
.489	w	0.877	f	.586	w	.459	m
.159	f	.384	w	.553	w	.382	w
.069	B_1f	.252	f	.416	f	.327	f
1.984	B_1f	.174	f	.317	w	.160	Fb
.764	w	19.963	f	.271	w	.013	m
.080	wc	.776	wb	.173	F	4.926	w
0.651	w	.585	f	.082	f	.864	f
.540	B_1w	.481	w	.020	w	.678	fb
.435	w	.181	wb	0.765	w	.559	m
29.796	w	8.593	wb	.729	w	.472	w
.680	f	.295	f	.577	f	.388	f
.190	m	.191	f	.525	f	.325	f
.057	B_1w	.020	wb	.436	F	.106	wc
8.733	wc	7.670	f	.239	w	.015	wc
.689	T_1w	.558	w	.198	w	3.665	ic
.589	w	.469	w	.067	w	.575	mb

TABLE I—*Continued*

λ	Descr.	λ	Descr.	λ	Descr.	λ	Descr.
3603.462	f	3600.192	f	3596.503	w	3592.880	w
.396	f	.111	i	.411	m	.786	m
.247	mbd	599.962	w	.272	w	.667	F
.110	mb	.839	w	.168	F	.570	i
2.987	m	.751	w	.067	m	.470	f
.879	m	.687	w	5.991	F	.361	f
.787	T_3m	.477	i	.906	i	.193	ib
.674	m	.391	f	.713	f	.085	w
.485	w	.243	w	.511	ib	1.985	f
.451	w	.136	f	.417	f	.878	ibc
.333	m	.046	i	.323	ib	.789	F
.224	m	8.867	f	.129	m	.696	f
.103	w	.790	m	.042	m	.595	w
.026	w	.699	f	4.899	wb	.505	ibc
1.892	m	.450	w	.787	w	.396	f
.778	m	.351	w	.659	w	.308	f
.676	w	.306	m	.587	m	.187	m
.611	w	.105	m	.486	w	.117	m
.458	w	.011	f	.403	m	0.988	m
.328	w	7.773	wc	.107	ib	.903	f
.296	w	.722	wc	3.966	w	.818	f
.222	f	.672	wc	.927	m	.769	w
.014	w	.434	w	.759	w	.676	w
0.899	wbc	.299	mb	.674	ic	.415	H'_1
.842	wc	.103	i	.510	fb	85.911	H'_2
.562	m	6.967	m	.294	ibc	3.935	H'_3
.467	w	.750	m	.074	f		
.432	w	.629	w	2.919	ibc		

Since series A_1 is the longest one known and affords excellent material for testing formulae, certain useful data pertaining to it have been collected in Table II. In the first column may be found the ordinal numbers of the lines; in the second, the wave-lengths in air; in the third, the frequencies or number of wave-lengths per centimeter *in vacuo;* and in the fourth and fifth columns, the first and second differences of the frequencies respectively. (First differences are *always* calculated from the mean positions of the resolved doublets.) The last two columns show clearly that the lines do not follow a smooth curve exactly.

DESCRIPTION OF SERIES

All except three of the lines measured between the first and third heads have been assigned to different series. By this classification it is found that both the spectrum from the first head and that from

TABLE II

No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.	No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.
0....	3883.402	25743.46	....	....	27...	3875.772	25794.15	....	....
1....	....	....	....	....	28...	missing	....	....	....
2....	....	....	....	....	29...	4.602	801.94	3.98	....
3....	....	....	....	....	30...	.004	5.92	4.21	0.23
4....	3883.191	25744.86	0.59	....	31...	3.371	10.13	4.23	.02
5....	.102	5.45	.75	0.16	32...	2.739	4.36	4.53	.30
6....	2.990	6.20	.88	.13	33...	.057	8.89	....	....
7....	.853	7.08	1.05	.17	34...	concealed	....	....	....
8....	.697	8.13	1.17	.12	35...	0.665	28.19	4.98	....
9....	.521	9.30	1.32	.15	36...	69.921	33.17	4.92	− .06
10....	.321	50.62	1.46	.14	37...	.180	8.09	5.17	.25
11....	.101	2.08	1.50	.04	38...	8.407	43.26	5.25	.08
12....	1.875	3.58	1.92	.42	39...	7.619	8.51	5.38	.13
13....	.587	5.50	1.88	− .04	40...	6.816	53.89	5.52	.14
14....	.305	7.38	2.02	.14	41...	5.991	9.41	5.62	.10
15....	0.999	9.40	2.19	.17	42...	.151	65.03	5.69	.07
16....	.670	61.59	2.28	.09	43...	4.300	70.72	6.09	.40
17....	.327	3.87	2.41	.13	44...	3.390	6.81	6.03	− .06
18....	79.964	6.28	2.56	.15	45...	2.489	82.84	6.18	.15
19....	.578	8.84	2.63	.07	46...	1.567	9.02	6.32	.14
20....	.183	71.47	2.86	.23	47...	0.626	95.34	6.41	.09
21....	8.749	4.33	2.97	.11	48...	59.670	901.75	6.62	.21
22....	.303	7.30	3.15	.18	49...	8.683	8.37	6.69	.07
23....	7.832	80.45	3.19	.04	50...	7.687	15.06	6.92	.23
24....	.351	3.64	3.38	.19	51...	6.658	21.98	6.95	.03
25....	6.843	7.02	3.51	.13	52...	5.622	8.93	7.12	.17
26....	.315	90.53	3.62	.11	53...	4.566	36.05	7.26	.14

TABLE II—*Continued*

No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.
54....	3853.487	25943.31		0.11
			7.37	
55....	2.391	50.68		.09
			7.46	
56....	1.285	8.14		.14
			7.60	
57....	0.158	65.74		.16
			7.76	
58....	49.008	73.50		.15
			7.91	
59....	7.839	81.41		.07
			7.98	
60....	6.677	9.26		.19
	.633	.53		
			8.17	
61....	5.468	97.44		.06
	.427	.69		
			8.23	
62....	4.250	6005.65		.14
	.206	.94		
			8.37	
63....	3.009	14.04		.17
	2.973	.29		
			8.54	
64....	1.753	22.55		.09
	.710	.87		
			8.63	
65....	0.479	31.20		.12
	.436	.48		
			8.75	
66....	39.189	39.93		.17
	.141	40.25		
			8.92	
67....	7.877	48.84		.14
	.825	9.18		
			9.06	
68....	6.540	57.92		.08
	.494	8.22		
			9.14	
69....	5.202	67.04		.08
	.147	.38		
			9.22	
70....	3.842	76.25		.19
	.785	.62		
			9.41	
71....	2.458	85.66		.10
	.404	6.02		
			9.51	
72....	1.064	95.15		.13
	.005	.56		
			9.64	
73....	29.650	104.79		.14
	.588	5.20		
			9.78	

No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.
74...	3828.211	26114.59		0.07
	.155	.97		
			9.85	
75...	6.770	24.42		.17
	.708	.84		
			10.02	
76...	5.307	34.41		.10
	.239	.89		
			10.12	
77...	3.823	44.56		.15
	.760	.98		
			10.27	
78...	2.322	54.82		.10
	.259	5.27		
			10.37	
79...	0.807	65.20		.10
	.746	.64		
			10.47	
80...	19.278	75.67		.13
	.213	6.12		
			10.60	
81...	7.732	86.28		.13
	.670	.71		
			10.73	
82...	6.170	97.00		.16
	.101	.45		
			10.89	
83...	4.587	207.87		.09
	.517	8.36		
			10.98	
84...	2.984	18.88		.06
	.923	9.32		
			11.04	
85...	1.382	29.91		.16
	.315	30.37		
			11.20	
86...	09.755	41.12		.32
	.690	.56		
			11.52	
87...	8.150	52.16		− .21
	7.941	3.57		
			11.31	
88...	6.440	63.96		.22
	.377	4.39		
			11.53	
89...	4.774	75.43		.12
	.696	.99		
			11.65	
90...	3.090	87.09		.17
	.013	.63		
			11.82	
91...	1.375	98.95		.05
	.308	9.42		
			11.87	

TABLE II—*Continued*

No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.	No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.
92...	3799.663	26310.79		0.16	110..	3766.322	26543.71		−0.11
	.587	1.33				.247	4.23		
			12.03					13.90	
93...	7.926	22.83		.07	111..	4.350	57.62		.13
	.852	3.35				.277	8.13		
			12.10					14.03	
94...	6.184	34.92		.11	112..	2.362	71.65		.09
	.104	5.47				.290	2.16		
			12.21					14.12	
95...	4.417	47.18		.17	113..	0.363	85.77		.10
	.351	.64				.290	6.29		
			12.38					14.22	
96...	2.639	59.53		.10	114..	58.348	600.03		.08
	.565	60.05				.287	.47		
			12.48					14.30	
97...	0.846	72.00		.06	115..	6.332	14.31		.13
	.771	.54				.263	.79		
			12.54					14.43	
98...	89.040	84.56		.09	116..	4.300	28.72		.04
	8.968	5.06				.227	9.24		
			12.63					14.47	
99...	7.228	97.19		.18	117..	2.258	43.20		.12
	.155	.70				.187	.70		
			12.81					14.59	
100...	5.390	410.01		.12	118..	0.206	57.78		.09
	.319	.51				.134	8.30		
			12.93					14.68	
101...	3.541	22.91		.06	119..	48.141	72.46		.02
	.465	3.47				.068	.98		
			12.99					14.70	
102...	1.685	35.90		.10	120..	6.075	87.17		.23
	.605	6.46				.004	.68		
			13.09					14.93	
103...	79.806	49.02		.13	121..	3.980	702.11		.02
	.732	.53				.910	.61		
			13.22					14.95	
104...	7.919	62.23		.10	122..	1.887	17.06		.05
	.842	.77				.817	.56		
			13.32					15.00	
105...	6.015	75.57		.09	123..	39.783	32.07		.10
	5.944	6.06				.716	.56		
			13.41					15.10	
106...	4.107	88.96		.11	124..	7.672	47.18		.01
	.030	9.50				.604	.66		
			13.52					15.11	
107...	2.180	502.49		.10	125..	5.612	61.92		.28
	.106	3.01				.444	3.14		
			13.62					15.39	
108...	0.240	16.12		− .03	126..	3.417	77.66		− .05
	.169	.62				.344	8.18		
			13.59					15.34	
109...	68.302	29.76		.42	127..	1.276	93.02		.07
	.247	30.16				.208	.51		
			14.01					15.41	

TABLE II—*Continued*

No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.	No. of Line	λ	$\frac{10^8}{\lambda_v}$	First Diff.	Second Diff.
128...	3729.130	26808.44		0.10	146..	3689.372	27097.32		0.04
	.064	.91				.327	.64		
			15.51					16.52	
129...	6.975	23.94		.11	147..	7.122	113.86		.05
	.906	4.43				.082	4.15		
			15.62					16.57	
130...	4.804	39.55		− .01	148..	4.872	30.43		− .01
	.732	40.07				.832	.73		
			15.61					16.56	
131...	2.639	55.18		.10	149..	2.622	47.00		.04
	.572	.67				.582	.29		
			15.71					16.60	
132...	0.457	70.94		.08	150..	0.370	63.60		.03
	.399	1.34				.331	.89		
			15.79					16.63	
133...	18.273	86.71		.06	151..	78.116	80.25		.00
	.210	7.16				.080	.51		
			15.85					16.63	
134...	6.083	902.56		.07	152..	5.864	96.88		.02
	.020	3.01				.830	7.15		
			15.92					16.65	
135...	3.884	18.47		.06	153..	3.614	213.54		.00
	.823	.94				.577	.80		
			15.98					16.65	
136...	1.682	34.46		.06	154..	1.368	30.20		.05
	.618	.91				.335	.45		
			16.04					16.70	
137...	09.470	50.52		.09	155..	69.099	47.03		.01
	.411	.94						16.71	
			16.13		156..	6.852	63.74		− .07
138...	7.252	66.64		.05				16.64	
	.191	7.08			157..	4.615	80.38		.04
			16.18					16.68	
139...	5.031	82.81		.01	158..	2.375	97.06		− .02
	4.967	3.27						16.66	
			16.19		159..	0.140	313.72		− .02
140...	2.805	99.03		.07				16.64	
	.750	.43			160..	57.910	30.36		.08
			16.26					16.72	
141...	0.575	7015.31		.05	161..	5.678	47.08		− .09
	.520	.67						16.63	
			16.31		162..	3.455	63.71		− .16
142...	698.342	31.60		.04				16.47	
	.287	2.01			163..	1.256	80.18		.05
			16.35					16.52	
143...	6.104	47.97		.04	164..	49.055	96.70		− .01
	.053	8.34						16.51	
			16.39		165..	6.856	413.21		− .05
144...	3.863	64.36		.06				16.46	
	.813	.73			166..	4.668	29.67		.04
			16.45					16.50	
145...	1.622	80.80		.03	167..	2.477	46.17		− .05
	.570	1.19						16.45	
			16.48		168..	0.296	62.62		

the second consist of two series—one a singlet series, the other a doublet series. (The term "singlet" is here used, although the series are, in all probability, really composed of unresolved doublets near the heads.)

The A series.—The intense first head apparently consists chiefly of the series A_1. On leaving this head and going toward shorter wave-lengths one acquires the impression that the A_2 series gradually grows out of, and away from, A_1. The first component of A_2 appears as a close companion of the eighth line of A_1. Line 12 of A_1 seems to be slightly displaced, is more intense and less diffuse than any of the other preceding lines. The second component of A_2 first appears as a close companion to line 13 of A_1. Series A_1 is relatively much stronger in intensity at first than A_2, but its intensity decreases steadily until it vanishes completely for line 28. At λ 3880.4 the intensities of A_1 and A_2 are equal. From here onward, the intensity of A_2 steadily decreases until at the third head it becomes so weak as to be lost in the increased complexity of the band structure. A most striking anomaly occurs at λ 3872, where line 32 of the A_1 series is displaced from its expected position by about 0.02 A and is relatively very broad and intense. Here also the component lines of A_2 are replaced by a single narrow line of weaker intensity than its congeners and displaced by about 0.08 A from its predicted position. From the missing line (28) onward, the intensity of A_1 seems to increase slowly until at 39 it becomes the strongest series in intensity and maintains this rank throughout its length. Its absolute intensity from line 39 diminishes up to the third head where again it seems to increase steadily until line 67 is reached. Thence its intensity diminishes slowly but uniformly. It is difficult to follow the absolute intensity of a series on account of the many superpositions of other lines. From line 47 on, the ultimate doublet nature of A_1 is apparent. The first line actually resolved is 60. The separation of these components increases gradually up to line 76, whence it is fairly constant as far as line 87. Here we find a second anomaly similar to the first. The two components are widely separated—the wave-length interval having increased from about 0.07 A to 0.21 A—and both are relatively more intense. The first component is this time the broader, prob-

ably owing to the superposition of a line from the C series. In line 88 the former state of affairs is resumed, but from here onward the components seem to be more widely separated. At line 125 we again find an anomaly. The components are widely separated (about 0.17 A), whereas line 126 appears normal. The interval between the components now diminishes, at first slowly, then more rapidly, until at line 155 the components are unresolved, although the lines are apparently of doublet nature. Meanwhile at about line 150 the first differences of wave-length have reached a maximum and begin to decrease. The last lines of the series are relatively extremely faint, having been obtained only with exposures of four hours. Owing to the weak and uniform intensity of the lines from 168 onward and to their great number, it is impossible to trace this series farther. The superposition of lines from other series and the accompanying changes in intensity are noted in Table I.

The B series.—As in the case of the first head, the intensity of the second head seems to consist chiefly in that of the singlet series. But here the doublet series, apparently emerging from behind the singlet series and simultaneously increasing in intensity, seems to precede the singlet series. On account of the superposition of so many lines, nothing definite can be ascertained concerning the absolute intensity of these series. For the same reason no attempt has been made to trace series B_2 beyond the fourth head. A line of series B_1 is missing at λ 3863.0, and another is concealed by the third head. This series first appears double at λ 3838.7. The interval between the components increases steadily up to λ 3821.8. Here it suddenly decreases and the line at λ 3819.0 is single, relatively more intense, and displaced slightly from its predicted position. The next line consists of two widely separated components with an interval of about 0.45 A. The mean position of the components, however, corresponds very nearly to the expected position. In the next line the components are once more near each other, although from here onward they are distinctly, on the whole, about 0.02 A farther apart than below λ 3820.4. A noteworthy anomaly occurs at λ 3777.1, where instead of a doublet we find a single, diffuse, relatively more intense line displaced from its calculated

position. Again the following doublets seem to be farther separated, but only slightly so.

At λ 3718.9 we have a precisely similar phenomenon. In all three anomalies the displacement of the single line from its predicted position is toward longer wave-length. Also, the average interval between the single line and its adjacent neighbors is larger than one would expect, as if the series had been stretched out at this point. The interval between the components has already begun to decrease and, from here onward, it steadily diminishes until at λ 3679.2 the components are again unresolved. All the lines, and they are numerous, in the region immediately following are of almost equal intensity. Consequently, it is difficult to be certain that the lines actually do belong to the series to which they are ascribed. Lines at λλ 3649.5, 3635.0, and 3632.0 appear to be double, although only one of the apparent components may really belong to this series. Lines expected at λλ 3639.7 and 3633.5 have not been found.

The C series.—Beginning with the third head the structure becomes so dense and complex that no attempt has been made to disentangle all the series beyond this point. Consequently the C_1 series has not been traced back to its head, although, as will be seen later, there is reason to believe that it has its origin in the third head. Only the C_1 series has been traced. Just as the B_1 series is everywhere less intense than the A_1 series, so the C_1 series appears distinctly weaker than the B_1 series. Its first anomaly occurs at λ 3830.6, where instead of a single line appear two widely separated lines, the interval being about 0.46 A and their mean position being displaced slightly from its expected position. The duality of this series appears suddenly at λ 3826.2, where the components are separated by about 0.12 A. This interval, however, immediately shrinks to about 0.07 A. At λ 3808.1, the C_1 series is apparently single and superposed upon the first component of line 87 of series A_1. At λ 3786.8 the two components appear rather faint and widely separated. There seems to be a discontinuity of some kind. It is here also that the second branch, D_1' of the D series, appears. The components are again widely separated at λ 3780.8. At λ 3755.0 no line appears for series C_1. However, there is a wide doublet at

λ 3755.2 that may really belong to C_1, as it has not been ascribed to any other series. If so, it is greatly displaced from its predicted position. At λ 3734.3 we have an anomaly similar to those found in series B_1. At this point occurs a single line, relatively more intense, and displaced from its calculated position. At λ 3710, in which region series D_2' possesses an anomaly, no line appears to represent the C_1 series. From here onward the interval between the components decreases until at about λ 3680 the lines are unresolved. Another anomaly occurs at λ 3682, where, instead of having a close doublet, two single lines appear, separated by an interval of about 0.41 A. No lines above λ 3669 have been found for this series.

The D series.—This series has not been traced back to its head, but appears to belong to the fourth head, as Jungbluth has already shown. The first few lines are faint and fuzzy. The double nature of the lines appears at λ 3829.4. The first section of D_1 ends at λ 3800.3. Here the series seems to be discontinued and the first section of E forces itself upon one's attention. The second section, D_1', appears at λ 3785.8 and continues for 14 lines. The components are distinctly farther apart than they were in the first branch. The 13th line, at λ 3768.9, is a very close doublet; and the 14th, at λ 3767.5, is a strong single line. There is some doubt as to whether these two lines should be ascribed to this series. Perhaps they should be replaced by λλ 3768.883, 3768.793, and λλ 3867.402, 3867.301. Yet the latter doublet seems to go better with the second section of E. This series is again distinguishable at λ 3762.9 and continues to λ 3743.6. At the end of this section the components converge rapidly, the last line being unresolved and seemingly slightly displaced toward longer wave-length. Almost immediately, at λ 3743.4, another section of apparently the same series (at least so far as one can tell by successive differences in wave-length) commences. This section is designated by D_2. The first line, a doublet, has been assigned to two series, E'' and D_2, although its intensity is *not* sufficient to justify this procedure. There seem to be two lines missing at λ 3724.2 and λ 3722.8. Yet a second branch, D_2', puts in its appearance at λ 3721.2. This section displays an anomaly at λ 3710.2 where instead of a doublet there is a relatively more intense single line slightly displaced from its

calculated position toward longer wave-length. From here onward, the components are scarcely resolved. At λ 3702.5 the line appears to be actually single, although the next line is double. The following line is missing. The third branch, D''_2, however, immediately appears and can be followed to λ 3690.8. This line is apparently displaced toward longer wave-length. From here onward it is very difficult to extend this series, although at times it seems to reappear in single lines so situated relative to the A_1, B_1, and C_1 series as to lead one to such an inference.

The E series.—This series is first noticeable at λ 3800.3 at the point where the first branch of D ends. Although most of its lines are confused with or superposed upon others, it can be traced to λ 3780.3, where it is lost. A second branch appears at λ 3777.4. The components here are more widely separated than before. At λ 3774.9 they are separated by 0.19 A which is more than twice their separation in the first branch. They converge, however, and at the end of this section, at λ 3762.3, they are not more than 0.10 A apart. It is in this region that D''_1 is first found. The third branch of E first appears at λ 3754.5 and can be traced to λ 3742.1, where it is confused with a titanium line. No effort has been made to trace this series farther, most of its sections having been found incidentally in the study of the other series.

The series compared.—In order to show these series graphically Jungbluth's method of plotting the first differences in wave-length as ordinates against the wave-length of successive lines as abscissae has been adopted (see the diagram). To bring out the uncertainty with regard to the assignment of lines to series, due to the "perturbations" referred to by Deslandres, the successive points have been connected by straight lines instead of smooth curves. The scale has been so chosen that practically every perturbation shown is real and cannot be due to errors in measurement. As would be expected, the graphs representing the series A_1, B_1, C_1, D_1, and E are very similar, but not parallel throughout their entire lengths. They clearly supply another argument, besides that of relative intensity, for assigning the C_1 and D_1 series to the third and fourth heads respectively. The very existence of an E series suggests the possibility of a fifth head to the 3883 band. This graph also affords

the chief argument in favor of combining the different sections of series in the manner chosen. One very noticeable fact is that the perturbations within the series increase as one passes from A_1 to B_1 to C_1, etc.—that is, from series of the first head to series of the second head, and so on. A second noteworthy fact is that perturbations in the A_1 series seem to be represented also in the B_1 series, but at longer wave-lengths. For the A_1 series these particular regions are designated by the letter P; for the B_1 series, by Q; and so on. Corresponding perturbations on the different series are designated by the same subscript. P_0 marks the first anomaly of the A_1 series. Its second anomaly is seen at P_1. Corresponding to this are the regions Q_1 and R_1 on B_1 and C_1 respectively, both of which represent the first anomalies noted in these series. S_1 probably does not correspond to P_1, Q_1, and R_1, but is largely due to the fact that in this region the doublet nature of the lines of the D_1 series first appears. P_2 represents an apparent discontinuity in series A_1. In this region all three series A_1, B_1, and C_1 are confused with or superposed upon one another. Corresponding to P_2 we have Q_2, representing the second anomaly in series B_1, and R_2, implying a discontinuity in series C_1. At S_2 there is a gap in series D_1 and it is in this region that E first appears distinctly. At P_3, the third anomaly of series A_1 is shown. Corresponding to it is the perturbation at Q_3 where the B_1 and A_1 series are apparently exactly superposed. R_3 represents a change in the character of series C_1, and also the region where one line is missing. At S_3 there is an anomaly in series D_1 as well as a gap. U_3 represents a region in E where apparently two lines are missing. It should be noted that this region is not wide enough for three missing lines nor small enough for two to fit the graph smoothly. We have the third anomaly of series B_1 at Q_4. R_4 represents a similar phenomenon in series C_1 followed immediately by the exact superposition of C_1 and B_1. S_4 designates a change in character and very abrupt break in D_1. In fact, owing to the sudden change in character at this point and the anomalous position of the first line of D_2 relative to the last line of D_1'', there is reason to believe that D_1'' and D_2 are not actually sections of the same series. Yet this graph indicates that they are closely related in some way. U_4 simply represents the region where

E″ first appears. Q_5 designates an actual perturbation in B_1. R_5 is a region in C_1 where a line is missing, and S_5 represents two missing lines in D_1 while E has come to an end. Probably the members of this last group are not so intimately connected. The missing line in C_1 is doubtless related to the anomaly in D_1 at that region, and the missing lines in D_1 to the anomaly in B_1. R_6 represents irregularities in series C_1 while S_6 designates an anomaly in series D_1.

THE TAILS

In Table III are given both King's values and those obtained in this investigation for the wave-lengths in air of the so-called tails.

King's grouping has been retained, but the separate tails have been denoted by subscripts which increase with increasing frequency. The intensity of group I is on the whole weaker than that of group II. Nothing can be said with regard to the relative intensity of group III on account of its position between two bands. The relative intensity of the tails within each group has been indicated by the same notation as used in Table I. A detailed description of the so-called tails follows.

TABLE III

Symbol		King's λ	λ in I.U.	Description
III	T_1	3658.34	3658.09	F
	T_2	3629.06	3628.69	f
	T_3	3603.12	3604.10	ib
II	T_4	3465.69	....	
	T_5	3433.17	3432.99	i
	T_6	3405.04	3404.85	I
	T_7	3380.58	....	
	T_8	3360.27	3360.01	m
	T_9	3340.64	3340.35	w
	T_{10}	3322.40	....	
	T_{11}	3296.48	3296.31	f
I	T_{12}	3203.84	3203.53	f
	T_{13}	3180.58	....	
	T_{14}	3160.32	3159.94	w
	T_{15}	3143.06	3142.60	w
	T_{16}	3128.00	....	

T_1 is extremely faint even on a negative obtained from a four-hour exposure. The head appears like a diffuse line, and there is a very faint blurr in the contiguous region on the side of longer

wave-length. The spectrum is too faint to tell definitely whether it is or is not a band structure.

T_2 is also faint. In appearance it closely resembles T_1 but is stronger in intensity. Its band structure, if it possesses one, appears only as a smeared blurr.

T_3 is not believed to be a band structure. It consists of a relatively intense, broad, diffuse line with a doublet close to it on the side of shorter wave-length and a single medium line close beside it on the side of longer wave-length. The lines in this region certainly do not form a series emanating from this so-called tail. This has already been noted by Ritz.

T_4 under high dispersion exhibits no band structure. It consists of a medium, diffuse, broad line with a blurr on each side and also two or three fine lines.

T_5 is one of the tails that is distinctly a band structure. Its head is relatively intense and sharp, shading off toward the red. A few of its lines have been measured and are given in Table IV.

TABLE IV

T_5	T_6	T_5	T_6
3432.99	3404.85	3433.81	3405.50
....	....	.91	.67
3.11	.95	4.03	.86
.20	5.01		6.06
.31	.09		.30
.45	.17		.56
.57	.26		.85
.63	.37		7.18

T_6, like T_5, is truly a band spectrum. A number of its lines have been measured, and they seem to form a series. These wave-lengths are also given in the same table. The enormous number of fine lines which form the background in this region precluded the possibility of tracing the series to greater distances from the edges of T_5 and T_6. For the same reason, the intention of reproducing the original negatives of all of the tails, by the half-tone process, had to be abandoned.

T_7 exhibits no band structure under high dispersion. It seems to consist merely of two very close single lines superposed upon a foggy background.

T_8 resembles a band only in so far as it has a clear background on the more refrangible side of its edge and shades off toward longer wave-lengths with the ordinary channeled gradation of intensity. This shaded region is very narrow and photographically continuous. Owing to the extremely large number of lines in this part of the spectrum it is impossible to state positively whether T_8 is a band or not. Certainly no line series can be traced from it.

T_9 suggests band formation only in the very restricted sense of T_8. It seems to have a second head very close to the first one.

T_{10} does not exist as a band structure. The region here consists of a foggy background upon which numerous lines are superposed, but there is no sign of a true tail.

T_{11} looks somewhat like a real band, although no series lines can be distinguished because of the general blurr. The head has the appearance of a diffuse doublet that is unresolved.

T_{12} may be a band, but it does not possess channeled intensity. The region adjacent to the head on the side of longer wave-length apparently consists of about ten lines of equal intensity packed so closely as to be barely distinguishable.

T_{13} consists merely of a very close doublet in the midst of a cluster of fainter lines. High dispersion gives no evidence of any band structure whatever.

T_{14} and T_{15} look a little like bands in the very restricted sense of T_8. No series can be followed to the apparent edges. T_{14} may be due simply to a collection of unrelated lines which suggest a band formation as a consequence of the relatively clear background on the more refrangible side of the group. T_{15} is characterized by possessing a doublet at the edge.

T_{16} is not believed to be a true band. Its edge appears to consist of a broad diffuse line superposed upon a single line. Just to the side of longer wave-length of this composite line is a blurred region of uniform intensity in which several discrete lines are imbedded.

DISCUSSION

The deviation from Deslandres' law for band series is clearly illustrated by the diagram. In each series the first differences in wave-length attain a maximum and then diminish. The suggestion

that each maximum is really the point of intersection of two independent series, running in opposite directions with approximately equal curvatures (as has been found to be the case in a number of other band spectra), merits consideration, but we have not been able to find any evidence in its favor in the series under investigation. Deslandres has ascribed a fifth head, at λ 3852, to the 3883 band. There seems to be a great deal of doubt as to the reality of this fifth edge. The existence of the E series certainly suggests the possibility of a corresponding head. Nevertheless, if one calculates the position of the hypothetical edge by extrapolation according to Deslandres' law, no positive indication of a real head can be found on our negatives. However, there are blurred areas in this region, one of which may either constitute a fifth head or, with equal probability, be due to the superposition of the many series.

The fact that two series of lines have been traced from the first head and also two similar ones from the second head lends support to Deslandres' idea that each head gives rise to the same number of like series. Thus it would appear that the total structure of the 3883 band consists of two series—one, a "singlet" series; the other, a doublet series—for each head.

TABLE V

Head	Tail	Ratio	Ratio
3590.52......	3203.84	1.12069	1.12077
3585.99......	3180.58	1.12746	(1.12755)
3584.10......	3160.32	1.13409	1.13418
3883.60......	3465.69	1.12059	(1.12063)
3871.59......	3433.17	1.12770	1.12772
3861.91......	3405.04	1.13417	1.13422

King's conclusion that the so-called tails are actually the ends of series emanating from the heads was based largely upon the correlation of the heads and tails and the ratios of their wave-lengths given in Table V. The first three columns of the table contain King's data,[1] and the fourth column, the ratios calculated from the wave-lengths of the authors. The numbers in parentheses were obtained by using, in each instance, the wave-length of the strongest

[1] *Astrophysical Journal*, 14, 326, 1901.

line in the region of the supposed tail. The concordance of the third and fourth columns seems to indicate that the present writers were measuring the same lines and edges as King. On the other hand, the brief description of the characteristics of the tails, given in an earlier section, shows conclusively that the tails (at least as we obtained them on several negatives) are far too unlike to justify any conclusions which may be drawn from the apparent agreement between the corresponding ratios in the two sections of Table V. Several of the tails may be due to the superposition of series of fine lines which form the background of the entire spectrum, and, if this be true, it would account for the apparent regularity in the spacing of the tails. By burning a carbon arc in oxygen and comparing the negatives with the spectra of the like arc in air, we found that the background was noticeably different in several places. As might be expected, the band groups at λλ 3590 and 3883 were not eliminated by the oxygen atmosphere because it was impossible to remove all traces of nitrogen from the carbon rods. On the other hand, the spectrum of an arc between copper electrodes in carbon-free nitrogen brought out strongly the third and fourth cyanogen groups but not the tails. Unfortunately the experimental conditions did not admit of making exposures of several hours' duration and therefore the absence of the tails proves little or nothing. We desire, however, to lay special emphasis on the opinion that it would be quite useless for any of our successors to attempt to clear up the matter by again studying the ordinary carbon arc in air. The spectrum of the region of the tails must, if possible, be obtained in such a manner as to get rid of the background. Perhaps this can be accomplished either by the method used by Grotrian and Runge[1] or by using the "active modification" of nitrogen as Strutt and Fowler[2] have done.

With regard to the first and second cyanogen groups King says: "Taking the successive heads of these two bands and locating their tails by means of the ratios, we find that the tails belonging to the first three heads of the 4216 band should be at λλ 3762, 3722, and 3684." Since these hypothetical tails belong to the three most intense heads of the 4216 band, we should expect them to be of greater intensity than either of the tails T_1 or T_2, which King has

[1] *Loc. cit.*

[2] *Proc. Roy. Soc.*, A **86**, 115, 1911.

ascribed to the fourth and fifth heads. They should therefore appear clearly on negatives which show the last two tails. A careful examination of such negatives (taken with high dispersion and resolving power) failed to reveal the slightest traces of tails in the first two regions. At λ 3683.8 there is a structure which may be a tail but it is even weaker in intensity than T_1. Taking all the evidence into consideration, we are forced to the conclusion that King's correlation of heads and tails is not valid.

Let us now consider Jungbluth's combinations of the heads and tails. He was led, by a study of curves similar to those in the diagram, to the conclusion that the tail belonging to the A_1 series was at shorter wave-length than the tail pertaining to the B_1 series. For the series from the least refrangible head attained its maximum first difference and minimum terminal intensity later in its course than the series from the next head, and so on. Consequently, he joined the A_1 series to an assumed tail at λ 3579, B_1 to T_3, C_1 to T_2, and D_1 to T_1. The tail of the shortest wave-length was purely hypothetical, its position being predicted by extrapolation based on certain relations found in the three other cases. As to the B_1 series, the present investigation has failed to verify the existence of a tail at λ 3604. Furthermore, the diagram indicates (in our opinion) that the B_1 graph does not bend around fast enough to meet the axis of abscissae at a point having as great a wave-length as T_3. If these statements be admitted, then Jungbluth's correlation is destroyed, since they leave no data by means of which to calculate relations which might support his arrangement. Moreover, although series C_1 can be produced to meet T_2 in a fairly smooth curve, certainly the trend of the graph representing series D is not such as to lead to T_1 smoothly. As mentioned above, series D presents a definite discontinuity at λ 3743.5. Here, the position of the first line of branch D_2 with respect to the last line of branch D_1 suggests at once the possibility that D_2 may represent the sudden reappearance of the doublet series corresponding to D_1. Indeed, it may well be that these series do not exist actually as continuous series but are really composed of discrete segments each of which is a true series in itself, usually beginning and ending with a discontinuity of some sort.

P. Weiss[1] has already called attention to the fact that Jungbluth's arithmetical progressions for both the lengths of the series and also the ratios of the wave-lengths of the heads to those of the tails have but slight significance as arguments in favor of any intimate connection between the heads and tails. The present work confirms and strengthens this adverse criticism. However, Jungbluth's correlation of heads and tails, as contrasted with King's arrangement, is probably correct if the series actually converge to tails. Jungbluth was also the first to state that the maxima of the first differences in wave-length of the series form an arithmetical progression. The degree of accuracy to which this holds is shown in Table VI. The first column gives the series; the second, the maximum first differences; and the third, the successive differences between the numbers in the middle column. The values of the maximum intervals were obtained from the diagram and are, therefore, only approximate.

TABLE VI

Series	Max. Interval	Difference
A_1	2.252	
		0.260
B_1	1.992	
		.240
C_1	1.752	
		.248
D_1	1.504	
		(0.240)
E	(1.264)	

In conclusion, it seems appropriate to review very briefly all the salient points in the evidence bearing on Thiele's hypotheses. The heads and tails in the cyanogen spectrum have been correlated in two different ways by King and Jungbluth. The arrangement proposed by the former is destroyed by the facts that some of the tails certainly do not exist and that there are marked differences in character among the remaining structures which may perhaps be looked upon as tails. The correlation suggested by the latter is practically without foundation for the following reasons: (*a*) the tail at λ 3604, which is necessary for the formation of the arithmetical progressions, has an extremely doubtful existence; (*b*) the tail at λ 3579 is purely

[1] *Astrophysical Journal*, **35**, 79–83, 1912.

hypothetical; (*c*) the more refrangible ends of the graphs for the series A_1, B_1, C_1, and D cannot all be extended so as to meet the axis of wave-lengths at the points marking the tails; and (*d*) Jungbluth's arithmetical progression for the lengths of the series (if admitted) can be accounted for by elementary calculations based on the approximate laws of Deslandres. Two of the tails, T_5 and T_6, possess a finite number of resolvable lines up to their edges and hence they must be looked upon either as ordinary bands shading off toward the red—in which case two more tails drop out of the list—or as tails which do not conform to Thiele's hypothesis concerning an infinite number of lines at the convergence wave-length. Furthermore, in the preceding paper[1] by the senior author it was shown that Thiele's "phase," c, is not constant for the a and δ series of the λ 5165 carbon band, and that the A_1 and A_2 series of the third cyanogen group do not constitute the positive and negative branches of a complete Thiele series. It appears, therefore, that the properties of band spectra implied by the formula $\lambda = f[(n+c)^2]$, and stated very unambiguously in words by Thiele, are not realized in nature.

On the other hand, the generalized conception of conjugate heads and tails is not invalidated by the preceding arguments. The experimental part of the investigations of King, of Strutt and Fowler, and of Geuter show conclusively that the same source may simultaneously radiate bands, or bandlike structures, which shade off in opposite directions. Also the occurrence of maxima in the first differences of wave-length in the cyanogen (or nitrogen) spectrum, and in the arc spectrum of phosphorus,[2] is usually considered as constituting an additional argument in favor of some general connection between heads and tails. Nevertheless, the facts that such maxima have not been found in the majority of band spectra and that, when maxima are observed, the series apparently vanish

[1] After this paper appeared in print its author received letters from Professor J. A. Anderson in which it was stated that he had tested the new method of calculating c on two bands, near $\lambda\lambda$ 5327 and 5500 in the absorption spectrum of iodine vapor, and found that the phase was "very far from constant" for the region of the head of each band. The number of lines used was from 50 to 70 and the wave-lengths did not have errors as great as 0.002 A.

[2] P. Geuter, *Zeitschrift für wissenschaftliche Photographie*, **5**, 1, 1907.

at a relatively short distance beyond the stationary points, *may* be interpreted as meaning that bands do not in general converge to tails, that the bands which fail to attain maxima constitute the normal mode of radiation, and that the few bands which do exhibit maxima are exceptional cases, the lines beyond the maxima corresponding perhaps to a condition of partial instability. Lastly, that line and band spectra may be intimately related in the general sense suggested by Thiele and others is borne out by the recent investigations of the band spectrum of helium by Fowler[1] and by Nicholson.[2]

SUMMARY

1. An exhaustive study of the 3883 band radiated by the ordinary carbon arc in air has been made. The wave-lengths of all the lines between λ 3883 and λ 3590 (about 1740), that could be measured even approximately, have been calculated in international units. The dispersion and resolving power used were 4/3 times as great as Jungbluth employed in the same region.

2. The intensity of each line relative to neighboring lines has been indicated.

3. Segments of seven different series have been traced, and the anomalies within them described.

4. The superposition of lines of different series has been noted.

5. The perturbations and anomalies occurring within these series are shown graphically, and an apparent relation between anomalies in the different series has been suggested.

6. All the tails of shorter wave-length than λ 3883, given by King, have been measured under high dispersion and their structure described.

7. A brief discussion of the present status of Thiele's hypothesis regarding tails has been given and the following conclusions drawn: (*a*) that there is no indication that any of the so-called tails are directly connected with the heads of the 3883 band; and (*b*) that, in agreement with the view of P. Weiss, the only experimental evidence in favor of Thiele's hypothesis is the occurrence of band structures shading off toward the red, and the existence of maximum intervals between the lines of the same series.

[1] *Proc. Roy. Soc.*, A **91**, 208, 1915.

[2] *Ibid.*, 432, 1915.

8. A band, hitherto unrecorded so far as we can learn, has been found and its series lines measured. (See Appendix.)

In conclusion the authors desire to express their sincere thanks to Professor H. Kayser for suggesting the problem and for his counsel during the work, and to Professor J. S. Ames for placing at the disposal of the senior author all the facilities of the Johns Hopkins physical laboratory.

APPENDIX

A NEW BAND SPECTRUM

In the course of the preceding investigation a carbon arc was burned in an atmosphere of nitrogen from which oxygen and carbon dioxide had been removed. On the negatives thus obtained appear two faint bands shading off toward the red, which, so far as we can find in the literature of the subject, have not been previously recorded. The exact source of these bands has not yet been investigated by us. The one at shorter wave-length could not be traced to its head on the negatives obtained. The other seems to possess a head at λ 3280.85. This head is blurred and confused and certainly does not consist of many lines—probably only one. The first few lines are double but soon close up and appear single. These singlets then break up into doublets of which the components of longer wave-length later develop into doublets also, thus forming a triplet series. The first band similarly seems to consist of a triplet series. A table giving the wave-lengths of the lines belonging to the second band follows.

TABLE

λ	λ	λ	λ	λ	λ
head 3280.85	3290.50	3302.31	3313.91	3326.42	3336.49
	2.51	.43	4.20	.88	7.05
.98	.59	4.88	6.74	7.42	.20
1.22	4.71	5.05	7.05	9.75	8.12
.34	.83	.21	.38	30.28	9.46
.97	7.06	7.72	9.90	.93	.78
2.78	.18	.92	20.26	3.02	.86
.86	.23	8.11	.64	.11	40.09
3.97	9.55	10.62	.73	.72	
5.30	.67	.87	3.13	.81	
6.84	.77	1.10	.54	4.48	
8.58	302.16	3.64	.99	6.31	

Sloane Physical Laboratory
Yale University
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ON THE WAVE-LENGTHS OF IRON ARC LINES IN THE NEIGHBORHOOD OF THE CALCIUM H AND K LINES

By E. G. BILHAM

The question as to whether the wave-length of a given spectrum line is susceptible to slight changes depending upon the nature of the source producing it is of fundamental importance in spectroscopic measurements of precision, and has received considerable attention in recent years. A number of different aspects of the problem present themselves. Slight differences have been detected between the wave-lengths of certain lines in arc and spark spectra of the same element under normal atmospheric pressure.[1] Thus it seems reasonable to suppose that in some cases the wave-length of the radiation is a function of the electrical stress prevailing in the exciting mechanism. Since a large amount of the information derived from the astronomical applications of spectroscopy is dependent upon measurements of displacements of spectrum lines derived from sources whose electrical and temperature conditions we can, in most cases, only surmise, the importance of increasing our knowledge in this direction can scarcely be overemphasized.

Another interesting aspect of the question has to do with the effect on the lines in, say, the arc spectrum of a given pure element, of mixing with it a large or small amount of a foreign substance. As an example of an effect of this kind, attention may be drawn to the work of Dr. K. Burns,[2] who found that, in certain cases, the wave-lengths of lines due to substances occurring as impurities in the iron arc were not the same as those obtained from the carbon arc containing a salt of the metal under consideration. Thus, in the case of manganese, the wave-length of each line in the triplet 6013, 6016, 6021 was found to be about 0.030 A greater when determined from the impurity lines in the iron arc than that obtained by Kilby, using a manganese salt in the carbon arc. Similar displacements, but in the opposite sense, were also detected

[1] Bilham, *Phil. Trans.*, A **214**, 368, 1914.

[2] *Comptes rendus*, **156**, 1976, 1913.

in lines due to barium. Whatever may be the cause of the shifts, there can be no question of the importance of recognizing their existence. The practical interpretation of the phenomenon is well expressed by Burns: "Enfin, l'existence de cet effet montre qu'il n'est pas prudent de prendre comme étalons des lignes d'impuretés, en leur attribuant les longueurs d'onde trouvées dans les mésures faites dans d'autres conditions." Published results seem to indicate that shifts of this character are only met with in isolated, and apparently fortuitous, instances. In many cases careful measurements have failed to detect any evidences of displacements. Mr. W. J. Hall, of this College, has investigated the case of an iron alloy containing a known percentage (25 per cent) of nickel. Repeated measurements of the nickel lines in the spectrum, obtained from the arc between poles of the alloy, gave results agreeing with the wave-lengths obtained from pure nickel, within the limits of experimental error. The lines measured were for the most part comprised within the limits λ 3700 and λ 4200.

Professor Fowler has suggested to me the desirability of investigating what may be described as the inverse of the effect discussed above. The possibility presents itself that a very strong line due to an impurity may be capable of producing displacements of lines of the predominating substance in the immediate neighborhood. In order to test this point in a typical instance, a number of photographs were taken over the region λ 3700–4000, with a 10-ft. Rowland concave grating, using the third order. The carbon arc containing iron filings was used as the source of light. A second set of exposures was then made, in which the carbon arc was charged with a mixture of iron filings and calcium chloride, the other conditions remaining the same. By this means the lines H and K were obtained very strong and reversed, superimposed on the iron spectrum. A number of iron lines in the immediate neighborhood of H and K were then measured on all the plates, with reference to selected standards (indicated by a letter s), whose wave-lengths were assumed to be those obtained by Burns.[1] The lines measured were in most cases very suitable for accurate measurement. The dispersion in this region is approximately

[1] *Lick Obs. Bull.*, No. 247, **8**, 27, 1913.

1.85 A per millimeter, so that it could be considered possible to obtain an accuracy of 0.001 A. Each plate was measured in both directions, using a Hilger measuring machine provided with a parallel-wire eyepiece.

The results of the measurements are given in Table I. The first two columns contain the wave-length, intensity, and quality of each line as given by Burns. The mean wave-lengths calculated from the two sets of plates are given in columns 3 and 7. In each case the probable error was calculated from the formula

$$\text{probable error} = 0.6745\sqrt{\frac{\Sigma d^2}{n(n-1)}},$$

and the values are given in columns 6 and 10. Columns 4 and 8 contain the intensities estimated from the plates.

TABLE I

	Wave-Length (Burns)	Intensity and Quality	Fe Filings on Carbon				Fe Filings + $CaCl_2$ on Carbon				A−B
			Wave-Length A	I	No. of Plates	Prob. Error ±	Wave-Length B	I	No. of Plates	Prob. Error ±	
s	3925.945	3B	[3925.945]	2	4	......	[3925.945]	3	4	......	......
	3932.635	3A	3932.631	2	4	.001	3932.634	1	3	.002	−.003
	....	.....	3933.658	7	3	.001	3933.666	40R	4	.002	−.008
	3935.817	4−1	3935.817	2	4	.001	3935.816	2	4	.001	+.001
	3937.334	2A	3937.333	1	4	.001	3937.331	1	4	.001	+.002
s	3942.446	3A	[3942.446]	3	4	......	[3942.446]	2	4	......	......
s	3956.461	4A	[3956.461]	3	4	......	[3956.461]	3	4	......	......
	3961.534	1D	3961.531	4	4	.001	3961.535	9	4	.001	−.004
	3966.069	5A	3966.068	4	4	.001	3966.069	3	4	.001	−.001
	3966.626	5bB	3966.624	2b	4	.001	3966.616	2b	4	.002	+.008
	3967.426	4A	3967.427	2	4	.001	3967.429	2	4	.002	−.002
	....	.....	3968.474	4	3	.001	3968.475	30R	3	.002	−.001
	3969.263	7+B	3969.257	9	4	.001	3969.258	9	4	.002	−.001
	3970.394	2A	3970.393	1	4	.001	3970.387	1	4	.001	+.006
	3971.328	4A	3971.327	3	4	.001	3971.327	3	4	.002	.000
s	3977.747	5+1	[3977.747]	5	4	......	[3977.747]	4	4	......	......

The H and K lines were obtained on the first set of plates as impurity lines of moderate strength, being probably derived from the carbon poles. They were measured in a number of cases and their wave-lengths computed in order to compare them with the values obtained by measuring the reversals on the second set of

plates. The agreement may be considered satisfactory in the case of H, the values obtained being also concordant with that found by St. John,[1] viz., 3968.476. The K line, on the other hand, exhibits a displacement of 0.008 A toward the violet in the case where it occurs as impurity, as the term is usually understood. This result was somewhat unexpected in view of the close concordance of the wave-lengths obtained by St. John under a great variety of different conditions. His mean value for the wave-length of K is 3933.667. It appears, then, that the K line is susceptible to a shift of the kind described by Burns, whereas H is stable.

With regard to the iron lines, only two exhibit any marked tendency to undergo displacements. Of these, the line 3966.6, which shows a violet shift of 0.008 A, is very unsymmetrical, being shaded toward the blue. On this account the measurements are subject to a personal error in estimating the position of the maximum. The uncertainty would not be the same in both sets of measurements because in set B the line is buried in the wing of the H line, so that its lack of symmetry is obscured. The shift of 0.006 A in the case of the line 3970.4 affords some evidence that an adjacent heavy impurity line may be capable of affecting wave-lengths of other lines, and indicates the desirability of further work on this question.

The possibility of a purely *photographic* displacement must not be lost sight of. The presence of a violent perturbation in the film at a given point might conceivably have some effect at a short distance. Judging, however, from the fact that the iron lines nearest on either side to H and K exhibit no change of wave-length beyond the limits of experimental error, it would seem that such an effect is negligibly small, if it exists at all.

Imperial College of Science and Technology
South Kensington, London
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[1] *Astrophysical Journal*, **31**, 143, 1910.

THE SPECTRA OF CATHODE METALS

By PHILIP ELY ROBINSON

Many workers with vacuum tubes have observed lines of the metals composing the electrodes in the spectra of the gases in the tubes. These lines are strongest near the cathode, though in working with an induction coil they may also appear near the anode, especially if the inverse current is marked. A systematic investigation of this phenomenon was undertaken by Goldstein.[1] He found that the lines of many metals, fifteen in all, used as cathodes, appeared, but only if the gas in the tube was nitrogen, and that the brilliancy of the metallic spectrum was greatly enhanced by immersing the vacuum tube in liquid air. It seems worth while to record the preliminary results of some work along the same line.

The tubes first used were straight, having an internal diameter of 2 cm. The electrodes were from 15 cm to 20 cm apart. A quartz window on the side of the tube permitted a view of the cathode and its immediate neighborhood. It was quickly found that this window must be removed to the end of a side tube some 6 cm or more long to avoid its being obscured by the sputtering of the cathode. The final tube used, 1.2 cm in internal diameter, was H-shaped, with four electrodes of different metals, one in each branch. This made possible the investigation of four different metals under similar conditions. The cross bar of the H afforded an end-on discharge of the gas in the tube free from the cathode spectrum. The cathodes throughout were disks of metal nearly filling the cross-section of the tube. At first these disks had one large central hole. Later they were pierced by numerous small holes.

The source of current was an induction coil capable of giving a 15 cm spark. The voltage in the primary circuit was generally 18 volts, occasionally 24 volts. Variations in the voltage did not seem to affect greatly the production of the spectrum of the cathode. On the other hand, a condensed discharge was usually more effective

[1] *Physikalische Zeitschrift*, 6, 14, 1905.

in securing a brilliant cathode spectrum than a discharge where no condenser was used.

Early experiments indicated that the cathode should be thin. No evidence of metallic lines was obtained with cathodes 1–2 mm thick. From later experiments I believe a thick electrode might be used successfully if pierced by numerous small holes, as used in obtaining positive rays. It was first thought that the cathode should be thin, so that it might become hot. But though increased temperature appears to strengthen the cathode spectrum, it does not seem necessary for its production.

While the cathode spectrum is obtained with most ease in nitrogen, it can be produced in other gases as well. For obtaining the cathode spectrum there is a most favorable pressure which varies with the gas in the tube and with the material of the cathode. The range of pressure over which the cathode spectrum is easily excited is large for nitrogen. This probably explains in part why Goldstein should have found the cathode spectrum in nitrogen only. The cathode spectrum has been obtained in hydrogen, contrary to Goldstein's direct statement, in carbonic oxide, and with especial brilliance in oxygen.

The spectra have been photographed with a small quartz spectrograph made by Hilger, and the wave-lengths determined. The spectra were found to be the spark, rather than the arc, spectra. But it is particularly suggestive to note the selection of lines which appear and their relative intensities. A comparison of Table I with the tables in Kayser's *Handbuch*, **5**, will show that while in general only the stronger lines there given have appeared in these experiments, yet many strong lines have not, and many faint ones have been recorded. Furthermore, the selection of lines and their relative intensities depend upon the gas in the tube. Compare, for example, the intensities of the copper lines $\lambda\lambda$ 2770 and 2766 in oxygen and in carbonic oxide or in air. Note also the great intensity of λ 2370 in oxygen, while it was found to be wanting in both carbonic oxide and in air. It has not been definitely settled whether the converse case exists of lines that appear in carbonic oxide or in air, but yet are not to be found in oxygen. Lines are to be seen in the photographs of the cathode spectra in

air and in carbonic oxide that apparently fulfil this condition. For example, in the case of air there is a fairly strong line, intensity 3, which appears to coincide with the copper arc comparison line at λ 2260.6. This line is not to be found on the oxygen plate.

But four metals have so far been used as cathodes. Of these only copper was examined at all thoroughly. The results are summarized under the head of each metal and the gas in the tube.

COPPER

Hydrogen.—The cathode spectrum of copper was obtained in hydrogen, usually close to the cathode. Occasionally, when a condensed discharge was used, brilliant green flashes, showing copper lines vividly, occurred in the neighborhood of the cathode. On one occasion, after the tube had become well coated with copper, a fairly brilliant cloud, pale green in color, appeared about a centimeter or more in front of the cathode, i.e., toward the anode. Visually examined this showed the copper lines at $\lambda\lambda$ 5218, 5153, and 5106 with great brilliance. Photographed with a two-hour exposure, it showed a number of copper lines, while the hydrogen spectrum was strong. These copper lines are given in the table under H. The hydrogen was not quite free from CO and water-vapor, but nearly so.

Air.—In air the copper lines appeared easily, and much more brilliantly than in hydrogen. In the photograph the nitrogen bands obscured some of the copper lines, making it uncertain just which copper lines were present. Exposures of equal length yielded more copper lines than appeared in hydrogen, corresponding to the greater intensity of the cathode spectrum. In the table under N only such copper lines are noted as were definitely identified. With three exceptions these comprise all the lines of wave-length shorter than 235 $\mu\mu$ that appear on the plate. These three exceptions apparently coincide with lines in the adjacent copper arc comparison spectrum.

Carbon compounds.—Occasionally through accidental heating of the sealing-wax used to seal in the electrodes, carbon spectra became predominant in the tube. The copper lines were readily obtainable therein. As with nitrogen, the carbon bands greatly

interfered with identification of the copper lines present. Only such as were certainly present are given under C in the table. Nitrogen bands are not apparent in the photograph, though they are to be expected from this source.

TABLE I

LINES OF COPPER CATHODE IN VARIOUS GASES

(Only lines of wave-length less than 331 $\mu\mu$ are given)

λ	Intensity						λ	Intensity						λ	Intensity				
	a	*s*	O	N	C	H		*a*	*s*	O	N	C	H		*a*	*s*	O	N	C
3308.1	8	7	3	3	..	..	2489.7	..	8	7	..	..	..	2263.9	..	3	4	3	2
3279.9	2	3	3	3	3	2	86.6	..	4	4	..	..	..	55.1	..	2	3	..	..
74.1	10	8	10	10	10	10	86.0	..	4	4	..	..	..	49.1	..	3	3	..	..
47.7	10	10	10	10	10	10	82.4	..	5	3	..	..	..	47.1	4	7	9	3	..
08.3	4	1	1	4	..	..	78.4	..	3	5	3	10	..	44.3	1	1	2	5	2
3194.2	4	6	3	5	2	1	73.5	..	3	5	..	3	..	42.7	4	7	9	1	..
3094.1	2	3	1	6	?	..	68.6	..	8	5	..	..	..	30.2	8	3	5	4	1
10.9	4	3	3	7	3	..	58.9	2	1	2	..	..	..	28.9	4	4	5	—	..
2961.2	6	5	5	8	4	3	51.9	..	1	1	..	..	..	27.8	8	2	2	3	..
2824.5	..	6	9	7	P	3	44.5	..	5	5	..	..	..	25.8	6	2	3	4	1
2769.9	1	8	7	—	2	..	41.7	..	6	7	8	4	..	24.9	..	2	2	..	..
66.5	6	2	4	6	6	3	36.0	..	5	3	..	..	..	18.2	2	6	7	2	..
22.0	..	4	3	..	..	..	33.7	..	3	3	..	..	..	15.4	6	3	4	}3	..
19.0	..	6	5	..	..	..	30.6	..	4	3	..	..	..	14.8	8	3	4		..
13.8	..	8	6	..	..	..	28.4	..	3	3	..	..	..	12.9	..	1	3	—	
03.5	..	9	3	..	..	..	24.7	..	6	4	..	..	..	10.4	2	5	7	1	..
01.3	..	10	5	..	..	..	12.5	..	5	3	..	..	..	2199.8	8	3	5	3	..
2689.6	..	10	5	..	..	..	03.6	..	8	8	..	..	..	95.9	..	3	4	..	..
66.6	..	6	3	..	..	..	00.2	4	6	8	..	..	..	92.4	2	5	7	2	..
18.5	10	8	7	8	9	5	2392.7	8	4	3	7	..	..	89.7	2	5	6	..	4
00.5	..	10	6	..	..	..	76.5	..	5	5	..	..	..	81.8	4	1	3	3	2
2599.1	..	8	2	..	..	..	70.0	6	10	10	—	—	..	79.4	4	5	7	4	2
90.8	..	5	3	..	..	..	56.7	4	6	5	..	..	..	75.1	..	3	3	..	..
53.3	1	2	2	..	..	..	55.2	..	4	3	..	..	..	65.1	4	1	3	4	..
45.1	..	10	10	..	..	..	45.6	2	2	3	..	..	..	61.4	..	1	2	..	..
29.6	..	8	7	..	..	..	36.3	..	4	2	..	..	..	51.9	..	3	1	..	..
26.8	..	5	4	..	..	..	03.2	6	4	3	..	..	..	49.0	2	4	5	..	..
23.2	..	4	3	..	..	..	2299.7	..	2	2	..	..	..	36.0	2	3	6	..	..
18.5	..	3	2	..	..	..	96.9	..	1	3	..	..	..	34.5	8	2	3	..	..
16.5	..	2	2	..	..	..	94.4	2	6	10	7	5	2	26.1	2	3	5	..	..
13.2	..	5	2	..	..	..	92.0	..	4	3	..	..	..	23.1	2	3	5	..	..
11.5	..	5	2	..	..	..	86.8	..	4	5	..	..	..	17.4	..	2	1	..	..
08.7	..	5	3	..	..	..	78.5	..	2	1	..	..	..	12.2	1	2	4	..	..
06.5	..	10	9	..	..	..	76.3	4	6	9	—	..	..	04.9	4	2	4	..	..
2492.2	6	6	9	9	6	2	65.5	..	2	2	..	..	..	....	..	..	..	..	..

a = arc spectrum of copper. P = present on lines of gas.
s = spark spectrum of copper. — = definitely not present.
O = oxygen. N = air from leak. C = carbon compounds. H = hydrogen.

Oxygen.—The copper spectrum was obtained with great brilliance in oxygen to the practical exclusion of all lines of oxygen.

In the table under O is given every line shorter than λ 3310 that showed on the plate, with three exceptions. The first exception, at λ 3135, intensity 2, was probably due to carbon as impurity. The second, at λ 2883, intensity 2, was due to CO. The third, at λ 2600, has not been indentified. The time of exposure was 1^h 50^m. The immediate neighborhood of the cathode was filled with a greenish-white vapor. On one occasion with a condensed discharge a brilliant green cone of vapor showing the copper spectrum vividly shot out to the rear of the large hole in the cathode, i.e, away from the anode, for a distance of 2 cm or more. It was not found possible to maintain this green cone for more than a few seconds at a time.

The results for copper are summarized in Table I. The wavelengths are given under λ. Under *a* and *s* are stated the intensities of these lines in the copper arc and spark, respectively, as given in Kayser's *Handbuch,* **5**. Under O, N, C, and H are given the intensities as they appeared in the cathode spectrum in oxygen, air, carbonic oxide, and hydrogen respectively, 1 denoting a line so faint as to be barely recognizable, and 10 maximum intensity.

ALUMINIUM

The aluminium cathode spectrum was only observed visually. The electrode was a wire about 2 mm in diameter ordinarily used as the anode in the tube in which copper was investigated. Aluminium lines were observed in the same gases as copper and with the same relative ease in production. The most favorable pressure for aluminium was less than that for copper. In oxygen there appeared on the rear of the electrode a small yellowish-green tuft, which showed the aluminium spark lines brilliantly. At the same time two other workers in the laboratory, who were investigating gaseous spectra, found aluminium spark lines from their electrodes in their photographs, Mr. Matthews in hydrogen, and Mr. Brooksbank in CO.

IRON

The iron cathode spectrum was observed first when the tube was filled with a mixture of air and carbon compounds coming from a leak caused by the melting of the sealing-wax. The addition

of oxygen greatly increased the intensity of the iron spectrum. In fairly pure oxygen and with a condensed discharge, a bluish vapor extended for several millimeters on both sides of the cathode. The spectrum of this vapor showed a great many strong iron lines with relative intensities indicating the spark spectrum. Lines of the gas in the tube were also present. An exposure of twenty minutes was sufficient for a strong photograph.

Oxygen is a gas which disappears rapidly in a vacuum tube. A constant supply of it is necessary to keep the tube running continuously. Yet with both copper and iron, when the cathode spectrum had been established, a much smaller supply of oxygen was necessary.

SILVER

Very little was done with silver. What were apparently spark lines were observed visually in air.

The results may be roughly summarized by saying that conditions favorable to the sputtering of the cathode were found to favor excitation of the spectrum of the metal of the cathode.

In closing it is a pleasure to thank Professor A. Fowler, at whose suggestion and in whose laboratory these experiments were carried out, for granting me the facilities of his laboratory.

Royal College of Science, London
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